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BERNARD GATES

A Pitch-Class Set Space Odyssey, Told by Way of a Hexachord-Induced System of Genera

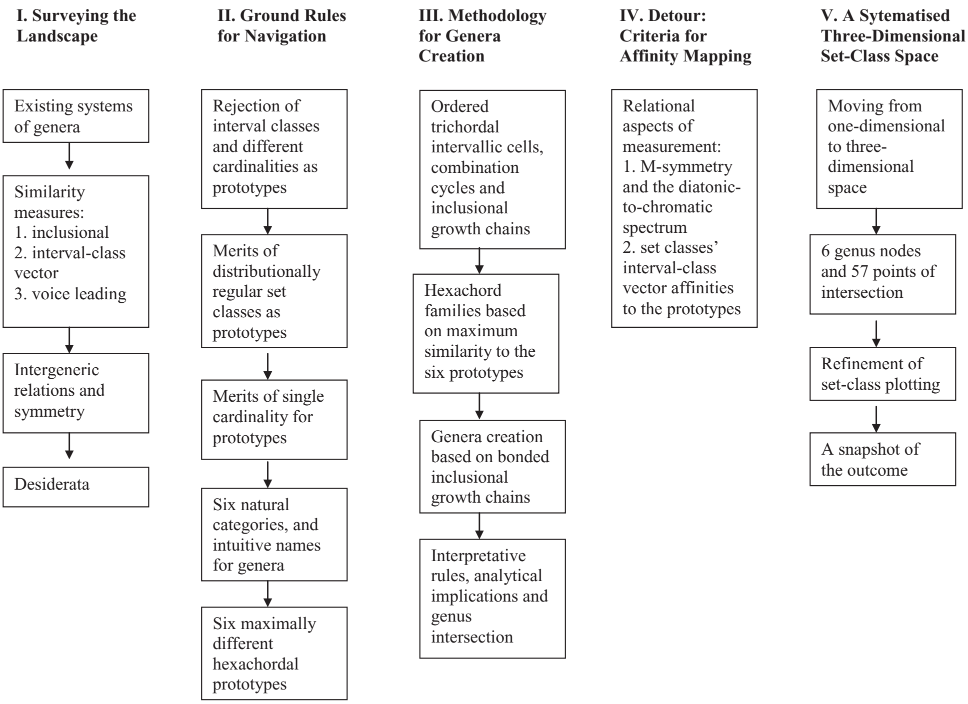
I. Surveying the Landscape, Setting the Scene

In this article, I aim to explore how a new system of pitch-class set genera could be formulated and graphically illustrated. Taking the systems of Allen Forte, Tore Eriksson and Richard S. Parks as a starting point, I briefly examine their approaches and limitations and suggest a method of moving forward from their seminal work. I present this study metaphorically as a 'space odyssey', traversing a number of stages whose narrative journey is mapped out in Fig. 1.

To date, the only published pitch-class set genera system to incorporate significant elements of intersection between its genera has been that of Allen Forte.[1] Two other methods, published around the same time, did not achieve this specific condition: Eriksson's 'regions' incorporated all set classes, but with only limited linkage between them, and Parks's theory relied solely on analytical context, with no preordained genera.[2] Forte's stance showed predictable insight and persuasion but was uncompromising in its approach. He stressed that his twelve genera were deliberately 'context insensitive' and purposely celebrated the consequent widespread location of familially linked set classes across them.[3] From 'very elementary and simple premises' he created a metaphorical 'spectrum' across his genera, from 'diatonic' (genera 11 and 12) to 'exotic' (genera 1, 2 and 3), through which subsets of familiar scale configurations and other categories such as aggregate or set-complex groupings were scattered or prismatically filtered.[4] Whereas Parks said of his own method that '[i]t is likely that each repertoire will require its own genus-models for achieving a "good fit" ', Forte emphasised that 'no such constraint exists in the case of my pitch-class set genera, which are abstract and unattached to any particular harmonic vocabulary'.[5]

It is now more than twenty years since Forte and Parks introduced their theories, and fourteen since the last flurry of interest in the use of their methods for the analysis of twentieth-century music.[6] This general neglect may be due in part, in Forte's case, to the need for a complex application of rules and 'difference quotients', and, in Parks's case, to the prerequisite need for his genera to be created anew for each discrete piece or repertoire. But a more fundamental obstacle stems from the structure of the theories themselves, their large number of genera and Forte's determination to initiate his genera inclusionally from a trichordal base.[7] These preconceptions lead to unwieldiness, on account of high

Fig. 1 Outline of the article



levels of genus membership. In Forte's case, the initial premises also resulted in high levels of intersection between genera, and also in such a skewing of set-class cardinalities that instances of trichords and tetrachords became restricted to between one and nine set classes per genus, while counts of pentachords and hexachords in many cases exceeded half of all those available. Further, this high saturation of pentachords and hexachords resulted in a 'disappointingly small' assignment of any of these set classes to just one genus.[8]

Building on ideas initially formulated in my PhD thesis, I am inclined to turn Forte's process on its head by initiating genera from a hexachord base, thereby relegating trichords to a more subsidiary generative role.[9] In this way I hope to instigate a method of genus categorisation that is akin to Eriksson's but with a more workable genus size, and that has a better numerical spread of set classes across the cardinalities and a more balanced degree of membership across the genera than Forte's system allowed, thus enhancing the possibility of coherent analytical practice.

What follows will be an attempt to elaborate these approaches while taking on board studies that have clarified the role played by similarity measures in the recognition of set-class difference and affinity. These latter inquiries have involved the development of a wide range of computationally derived similarity measures, typically consisting of indices of scaled values obtained through a comparison of pairs of numbers representing set classes. Some of these similarity measures have been based on interval-class vector analysis, such as Robert Morris's ASIM equivalences, Eric J. Isaacson's IcVSIM and Michael Buchler's SATSIM, while others have looked at subset and superset inclusional relations, such as David Lewin's REL, John Rahn's ATMEMB and Marcus Castrén's RECREL.[10] Subsequent comparative studies of these similarity measures by Ian Quinn, Thomas R. Demske and Art Samplaski have shown that similar taxonomic results can be produced from both types of procedure and that structural commonalities are evident between most, if not all, of the individual methods.[11] Multi-dimensional scaling, a dimensionality-reduction technique, has aided the visualisation of these set-class relations in terms of distance and how dissimilar (far apart) or similar (close together) they might be.[12] Allied to this practice is Quinn's concept of 'fuzzy' relations, which has defined set classes in terms of the degree of membership they possess in relation to prototypical sets within prescribed regions.[13] Further to this, cluster analysis has allowed a more refined classification of set classes into regions displaying these set classes' intrageneric affinity. Quinn's application of this method has allowed him to incorporate all set classes of cardinals 3, 4 and 5 into clusters corresponding to the six interval classes, a process that has produced results not dissimilar to those of Eriksson.[14] While the course of action adopted in this article follows a non-computational (yet still logical or algorithmic) path, the evidence from these scaled similarity measures, and from the ensuing comparative studies, will be invoked at a later stage as a means of confirming and to some extent fine-tuning the graphical representation of the proposed system of genera.

A different way of viewing similarity relations, based on voice leading, has recently been pursued by Joseph N. Straus, as well as Clifton Callender, Ian Quinn and Dmitri Tymoczko.[15] Little reference will be made to these studies in this article, however.[16] I will argue that since voice-led similarity is a concept primarily correlated to the smooth and efficient (by interval class 1 or 2) movement of parts, as is typically found in traditional Western forms of music, a different course should be followed when the primary concern is that *all* harmonic types and *any* intervallic form of melodic movement of parts (i.e. all displacements or transformations of one set class into another by any interval class) should be considered as being of equal potential value.[17] This latter course is desirable in that it allows us to embrace significant parts of the twentieth-century repertoire, where composers intentionally admitted the full gamut of dissonant harmonies (as well as consonant ones) and wider and less predictable intervals (as well as smoother ones). It will become apparent in due course that the intrageneric and cross-generic similarities suggested by this article are different in kind to voice-led similarities.[18] Straus's comprehensive arrays of ic 1 voice leadings for each cardinality, induced from set classes' prime forms, and Quinn's ic 1 array of tetrachords, induced from the ic 1 Fourier balance, both demonstrate an affinity dimension ranging from maximally compact and chromatically clustered to maximally even and anti-chromatic.[19] Consequentially, this affinity dimension inconveniently gathers most of the maximally dissimilar (in terms of interval-class vector) prototypical hexachords into the (terminal) maximally even camp. This runs counter to my preferred option for similarity progressions, which would proceed unilaterally, within each individual genus type, thus preserving the archetypal status of the prototypes while still allowing bilateral or multilateral affinities to emerge.

This article departs from previous studies in several ways. Fundamental to the enterprise is a system of genera which I first introduced in 1999 and which is based on six natural categories or scales and around six maximally dissimilar hexachords: the chromatic set class 6–1 (0,1,2,3,4,5), the diatonic 6–32 (0,2,4,5,7,9), the hexatonic 6–20 (0,1,4,5,8,9), the 'bichromatic' 6–7 (0,1,2,6,7,8), the whole-tone 6–35 (0,2,4,6,8,10) and the octatonic 6–30 (0,1,3,6,7,9).[20] Instrumental to the selection of membership in each genus is a system of set-class linkage based on inclusional relations proceeding from one cardinality to an adjacent cardinality, which I shall call 'inclusional growth chains' or 'IG chains'.[21] Following the examples of Eriksson and Quinn, I adhere to a hierarchical and reductional process, although the 'groups/pitch-class sets/set classes/genera' lineage will be taken one step further to the ultimate stage of affinity representation.[22] Inherent in this approach, however, will be one or two backtracking avenues of expansion. One of these will be the development of bonded inclusional growth (BIG) chain schemata as a means of creating genus membership. Others will be the creation of interval-class vector scalings for each genus as a means of establishing set-class affinity, as a measurement of distance, and for the establishment of intrageneric and intergeneric relations across the cardinalities. These scalings and intersections will help to locate all set classes in a set-class space, even those that have previously proved difficult to pin down.[23] Central to this endeavour will be a commitment to fundamental assumptions about the validity and reality behind pitch-class set or set-class equivalence, as well as a pragmatic stance on what characterises generic types and what might instinctively or implicitly be perceived to be resemblance within and between generic types. In this, I am following Quinn's belief in the theorist's capacity for developing ideas intuitively, although I lean rather more than Quinn does towards valuing and incorporating *ad hoc* solutions instead of rigidly and extensively argued abstract notions.[24] Aspects of symmetry will emerge as natural elements within the genera and will reappear as an overriding feature of the graphical representation of the genera in three-dimensional space. While Straus's arrays clearly have a spatial aspect to them, and Callender, Quinn and Tymoczko's 'quotient spaces' and 'global-quotient orbifolds' emerge naturally out of the forms of mathematical modelling that they are using – and, it might be added, while the multidimensional scaling of similarity measures translates theoretically into three or more spatial dimensions – my approach is deliberately non-mathematical and aims to create a more informal spatial model which can nevertheless show generic and distance relations between all set classes simply and at a glance, at once both comprehensive and comprehensible. Quinn has proposed that such a structure should incorporate overlapping regions based on contrasting genera and suggests a number of desiderata for such a design which I intend to adopt.[25] These desiderata can be summarised as the following key criteria (with my clarifications of Quinn's terms in square brackets):

1. that 'chord quality' [the quality which defines set classes and their relational location in 'quality space'] should take account of closely and distantly related sonorities [pitch-class sets] (p. 115);
2. that 'quality space' [a unified and regulated spatial model showing closely and distantly related sonorities or set classes] should be determined by similarity relations through the (computational) application of a numerical index (translatable as a spatial distance metric), and/or by a 'hierarchical taxonomy' of species [set classes] or genera (p. 115);
3. that theorists 'will present a relatively simple procedure that generates a whole system' (p. 115);
4. that the degree of similarity between sonorities [pitch-class sets] should relate to distances between them in 'quality space' (p. 115);
5. that set classes belonging to a single genus 'would lie near one another in quality space' (intrageneric affinities), and that the system as a whole would accommodate overlapping regions (intergeneric affinities) (pp. 115 and 121);
6. that 'quality space' should emerge from assumptions about the nature of 'chord quality', such as 'common tones' [pitch-class and pitch-class set affinity], inclusivity and combinatoriality (pp. 116 and 118);
7. that account should be taken of 'an *abstract* [generalised] notion of chord quality, in which quality is something that inheres in equivalence classes of chords' ['species' or set classes] such as transpositional, inversional and multiplicational equivalence and complementation, interval content [interval-class vector 'profile'], subset structure and transformational symmetries (pp. 119–21; emphasis in original);
8. that 'higher order taxonomic categories or genera [should be] organised around privileged, highly symmetric chord species' [set classes] or 'prototypes' which are 'quite distant from one another in quality space' (p. 121); and
9. that intrageneric and intergeneric affinities should be structured through qualitative closeness (similarity) or distance (difference) in 'quality space' to or from the designated genus prototypes (p. 121).

In its attempt to meet each of these stringent conditions, this exploration will make a progressive journey through the domains of set-class attribution and association in order finally to arrive at a three-dimensional representation of set-class space.

II. Setting Off with Ground Rules for Navigation: the Establishment of Six Categories and Their Hexachordal Prototypes

Classificational endeavours have often started with an examination of symmetrically and transpositionally invariant set classes built from the six interval classes, and of the larger scales or aggregate collections derived from them. My intention to demote smaller set classes as initial candidates for the creation of set-class genera may strike the reader as controversial. After all, the six interval classes themselves have traditionally been seen as fundamental to the process of generation and categorisation, dating back to Howard Hanson's extensional procedures (Fig. 2) and continuing through Eriksson's somewhat later investigation into 'maximising' the interval classes, Forte's superset extensions of his trichordal representations of the interval classes, Buchler's theory of i-set (saturation of an interval class) resemblances, Quinn's more recent consideration of 'intervallically constituted genera' and Justin Hoffman's interval-class displacement cycles.[26] But Eriksson has shown that natural categories or scales, as represented by Hanson's six 'great categories' and Eriksson's own six 'regions', can sometimes be induced from more than just one interval class.[27] It therefore seems necessary to make a distinction at the outset between interval classes' being of no particular use as generators but being of more general use as pointers towards genus membership. This distinction allows the significance of *relative* balance between the six interval classes to be brought into focus, as demonstrated within any initiatory set class's interval-class vector.

Quinn nominates all of the set classes in Fig. 2 as tentative genus prototypes for each of Hanson's six interval-class projections but ultimately opts for a smaller group of maximally even primary prototypes. These include a distinctive pattern of one primary prototype per cardinality per genus, that is, the 'bichromatic' 2–6 (0,6) from the ic 6 projection, the hexatonic 3–12 (0,4,8) from the ic 4, the octatonic 4–28 (0,3,6,9) from the ic 3, the diatonic 5–35 (0,2,4,7,9) from the ic 5, the whole-tone 6–35 (0,2,4,6,8,10) from the ic 2 and the chromatic 12–1 (0,1,2,3,4,5,6,7,8,9,10,11) from the ic 1.[28] I would like to distance myself from the prevalent view that these maximally even set classes act for genera in some unique way, and that the anti-prototypical extreme necessarily has to mean maximally compact set classes. A somewhat bizarre consequence of this assumption is that, as a matter of principle, just one maximally even set class per cardinality (together with its complement) might somehow typify each genus. Another consequence is that an opportunity to present intergeneric similarity relations is lost.[29] A more apposite set-class attribute for genus prototypicality surely ought to be distributional regularity, a feature concomitant with either inversional or transpositional combinatoriality or invariance. This quality would, as a consequence, admit prototypical candidates of minimal or equivocal evenness such as 6–20 (0,1,4,5,8,9), 6–30 (0,1,3,6,7,9), 6–7 (0,1,2,6,7,8) and 6–1 (0,1,2,3,4,5).

Fig. 2 Hanson's interval-class projections expressed as inclusional growth chains

2–1 (0,1)	3–1 (0,1,2)	4–1 (0,1,2,3)	5–1 (0,1,2,3,4)	6–1 (0,1,2,3,4,5)	7–1 (0,1,2,3,4,5,6)	8–1 (0,1,2,3,4,5,6,7)	9–1 (0,1,2,3,4,5,6,7,8)	10–1 (0,1,2,3,4,5,6,7,8,9)
2–2 (0,2)	3–6 (0,2,4)	4–21 (0,2,4,6)	5–33 (0,2,4,6,8)	6–35 (0,2,4,6,8,10)	7–33 (0,1,2,4,6,8,10)	8–21 (0,1,2,3,4,6,8,10)	9–6 (0,1,2,3,4,5,6,8,10)	10–2 (0,1,2,3,4,5,6,7,8,10)
2–3 (0,3)	3–10 (0,3,6)	4–28 (0,3,6,9)	5–31 (0,1,3,6,9)	6–27 (0,1,3,4,6,9)	7–31 (0,1,3,4,6,7,9)	8–28 (0,1,3,4,6,7,9,10)	9–10 (0,1,2,3,4,6,7,9,10)	10–3 (0,1,2,3,4,6,7,8,9,10)
2–4 (0,4)	3–12 (0,4,8)	4–19 (0,1,4,8)	5–21 (0,1,4,5,8)	6–20 (0,1,4,5,8,9)	7–21 (0,1,2,4,5,8,9)	8–19 (0,1,2,4,5,6,8,9)	9–12 (0,1,2,4,5,6,8,9,10)	10–4 (0,1,2,3,4,5,6,8,9,10)
2–5 (0,5)	3–9 (0,2,7)	4–23 (0,2,5,7)	5–35 (0,2,4,7,9)	6–32 (0,2,4,5,7,9)	7–35 (0,1,3,5,6,8,10)	8–23 (0,1,2,3,5,7,8,10)	9–9 (0,1,2,3,5,6,7,8,10)	10–5 (0,1,2,3,4,5,7,8,9,10)
2–6 (0,6)	3–5 (0,1,6)	4–9 (0,1,6,7)	5–7 (0,1,2,6,7)	6–7 (0,1,2,6,7,8)	7–7 (0,1,2,3,6,7,8)	8–9 (0,1,2,3,6,7,8,9)	9–5 (0,1,2,3,4,6,7,8,9)	10–6 (0,1,2,3,4,6,7,8,9,10)

I would argue that while generic, incrementally sequenced set classes such as Hanson's projections (see again Fig. 2) and Eriksson's maxpoint structures (his ex. 2) have been built upwards from the interval classes, they can just as plausibly be viewed as having been built *outwards* from a centrally placed platform of hexachords. Such an approach has generally been ignored as a source of generation, despite the potential benefit to be gained from hexachords' strategic standing at the centre of many systematically built symmetrical IG chains and their pivotal role in other types of relational IG chains as well, as will be shown later. Indeed, the use of 'famous' hexachords, that is, the all-combinatorial 6–1 (0,1,2,3,4,5), 6–32 (0,2,4,5,7,9), 6–20 (0,1,4,5,8,9), 6–35 (0,2,4,6,8,10), 6–7 (0,1,2,6,7,8) and 6–8 (0,2,3,4,5,7), together with the part-combinatorial 6–27 (0,1,3,4,6,9) and 6–30 (0,1,3,6,7,9), as possible genus representatives seems to be implied by Quinn in his discourse on hexachordal similarity relations.[30] Moreover, seven of these eight 'famous' hexachords are centrally placed in Eriksson's series of maxpoint set types and in Buchler's 'cyclic sets in 12 pc space', and six of them feature in Hanson's projections (see again Fig. 2).[31] Each hexachord in these sequences is strategically placed at its centre but is also in a sense terminal, since it occupies the point of reflection and symmetry between the smaller set classes and their larger complements.[32] If we consider the particular component of each projection that best matches its intuitive name, it becomes clear that hexachords predominate over other cardinalities. They are present in at least two of the projections, the (0,2) and (0,4), as the six-note whole-tone and hexatonic scales, and possibly in a third, since set class 6–7 (0,1,2,6,7,8) is the perfectly formed, all-combinatorial central element within the (0,6) 'bichromatic' projection. There is certainly a practical advantage in restricting prototypicality to a single cardinality so that operations and comparisons can work across the genera on a level playing field. The hexachords under consideration represent the only cardinality that exclusively epitomises all six of the tentative genera. They also specifically comply with certain basic cognitive characteristics of familial categorisation, share a neutral and common organisational level (i.e. they show equivalent cardinality) and present recognisable identities and mental images (i.e. they equate to 'pentatonic', 'diatonic', 'chromatic', 'hexatonic', 'whole-tone', etc.).[33] The inversionally or transpositionally invariant hexachords that have already emerged as part of Hanson's projections (Fig. 2), Eriksson's 'maxpoints' and Buchler's 'cyclic sets in 12 pc space' are 6–1 (0,1,2,3,4,5), 6–32 (0,2,4,5,7,9), 6–20 (0,1,4,5,8,9), 6–7 (0,1,2,6,7,8), 6–35 (0,2,4,6,8,10), 6–27 (0,1,3,4,6,9) and 6–30 (0,1,3,6,7,9).[34] All but one of these are representative of distinct but accessible and familiar scales or aggregates which can now be formalised by lending their names to five of the potential 'intuitive' genera. These are the chromatic genus (represented by 6–1), the diatonic (represented by 6–32), the hexatonic (represented by 6–20), the whole-tone (represented by 6–35) and the octatonic (represented by 6–27 and/or 6–30).[35] The remaining prototypical hexachord, 6–7 (0,1,2,6,7,8), represents the sixth, as yet unnamed genus: I am proposing the name 'bichromatic' for this potential genus, since its prime form (0,1,2,6,7,8) can be partitioned into two (transpositionally combined) chromatic segments a tritone apart, that is, (0,1,2)/(6,7,8) or, alternatively, into three chromatically spaced tritones, that is, (0,6)/(1,7)/(2/8).[36]

For the working out of further membership of these genera and as an ameliorative force for both facilitation and constraint, I therefore propose that the six hexachords offered earlier as representatives of the six 'intuitive' genera should henceforth act as primary genus prototypes for these six genera (Table 1). These correspond largely to Hanson's six projected hexachords, with one exception: my preferred octatonic representative would be 6–30 (0,1,3,6,7,9), rather than Hanson's (0,3) projected 6–27 (0,1,3,4,6,9). There are three reasons for my choice. Firstly, set class 6–30 has transpositional invariance, while 6–27 has neither transpositional nor inversional invariance. Secondly, 6–30 has fewer pentachordal subsets, three as against 6–27's five, an attribute which will ultimately act as a guard against high octatonic genus membership (as elsewhere will 6–20's one pentachordal subset, 6–7's two, 6–35's one, 6–32's three and 6–1's three). Thirdly, 6–30 has cyclically distributed elements of its prime form (i.e. a three-way combination of instances of ic 6 displaying the cyclic interval pattern 1–2–3–1–2–3), just as the octatonic scale (0,1,3,4,6,7,9,10) and three other octatonic subsets, 4–28 (0,3,6,9), 4–9 (0,1,6,7) and 4–25 (0,2,6,8), have cyclic interval patterns. Indeed, an examination of 6–30's prime form confirms that this prototypical octatonic hexachord also ideally 'contains' as subsets all of these three symmetrical tetrachords: (**0**,1,**3**,**6**,7,**9**), (**0**,**1**,3,**6**,**7**,9) and (0,**1**,**3**,6,**7**,**9**).[37]

To sum up, each of the six prototypical hexachords is distributionally regular, has a small number of subsets and has a transpositional and/or inversional redundancy built into its structure (all are all-combinatorial except 6–30). Each can nevertheless 'represent', if need be, each one of Quinn's maximally even primary prototypes from Hanson's projections, that is, set class 6–7 for 2–6 (10–6), 6–20 for 3–12 (9–12), 6–30 for 4–28 (8–28), 6–32 for 5–35 (7–35), 6–35 for itself and 6–1 for 12–1. Each is maximally dissimilar to the other five, and indeed to all other hexachords, in terms of the uniqueness of its interval-class vector content across two or more interval classes, allowing it to occupy a distant, lonely place in the set-class universe. Table 2 lists these

Table 1 Genus prototypical hexachords, their prime forms and interval-class vectors

Genus	Hexachordal prototype	Prime form	Interval-class vector
Hexatonic	6–20	(0,1,4,5,8,9)	[303630]
Bichromatic	6–7	(0,1,2,6,7,8)	[420243]
Octatonic	6–30	(0,1,3,6,7,9)	[224223]
Whole-Tone	6–35	(0,2,4,6,8,10)	[060603]
Diatonic	6–32	(0,2,4,5,7,9)	[143250]
Chromatic	6–1	(0,1,2,3,4,5)	[543210]

Table 2 The unique interval-class vector features of the six genus prototypes

6–20 [3**0**3**6**3**0**] (hexatonic)	highest ic 4	lowest ics 2 and 6
6–30 [22**4**22**3**] (octatonic)	2nd-highest ic 3, highest ic 6	evenly distributed ics 1, 2, 4 and 5
6–35 [**060603**] (whole-tone)	highest ics 2, 4 and 6	lowest ics 1, 3 and 5
6–32 [**1**432**50**] (diatonic)	highest ic 5	2nd-lowest ic 1, lowest ic 6
6–1 [**5**432**10**] (chromatic)	highest ic 1	2nd-lowest ic 5, lowest ic 6
6–7 [**4**2**0**2**43**] (bichromatic)	highest ic 6, 2nd-highest ics 1 and 5	lowest ic 3

unique features.[38] Clearly, these six prototypical hexachords display exceptional intervallic features that distinguish them as exemplars for their respective generic regions. As a manifest group, they conform wholly with, or equate closely to, those companies of set classes set aside as being special (and especially different) by several other theorists. These include Tenkanen's 'strangest' or 'cornerstone' hexachords (6–35, 6–20, 6–7, 6–30, 6–32 and 6–1), Joliffe's 'best discriminator' hexachords (6–35, 6–1, 6–7, 6–20, 6–32 and 6–30A/B), Eriksson's 'maxpoint' sets (whose hexachord representatives are 6–1, 6–32, 6–7, 6–20, 6–35 and 6–27), the pivotal hexachords in Buchler's 'cyclic sets in 12 pc space' (6–1, 6–35, 6–27, 6–20, 6–32, 6–7 and 6–30) and several of Quinn's tentative prototypes.[39] Quinn calls prototypicality 'the limit case' of a generic property that lies spatially in a 'remote' 'population center' as 'distant' 'landmarks'; Eriksson similarly places several of his maxpoint hexachords at the extremes of his regions in a diagram incorporating 'M-structure'; Tenkanen comes to the same conclusion, that set classes such as these occupy the farthest and remotest points in set-class space – that through their uniqueness they occupy distant bastions of differing typicality, thus satisfying key criterion 7.[40] The effect that these points of control can have on the spatial location of all other set classes will be visited in the final part of this study.

III. Moving On with a Methodology for the Foundation of a Balanced and Workable Society of Pitch-Class Set Genera: an Exploration of Ordered Trichordal Inclusional Cells and Combination Interval Cycles, Encounters with Six Hexachord 'Families' and the Establishment of a Lexicon of Bonded, Inclusionally Built Growth Chains

The journey towards grouping set classes into opposed but interlocking societies can start with a consideration of the role of trichords in creating identifiable groups before we move on to the more crucial role played by hexachords in establishing these larger communities. If the six interval cycles themselves are to be bypassed, as being of ambiguous use as progenitors, then ordered forms of the twelve trichordal set classes might offer a more productive point of departure and first frontier for this odyssey, since they at the very least have a clearer relationship to the six pilot genus regions than do the interval classes themselves. Ostensibly, a venture commencing from trichords might seem akin

Table 3 The 36 ordered trichordal intervallic cells (OTICs) having interval sums of between 2 and 12 (the 36 ways of summing two integers of between 1 and 11 to totals of between 2 and 12)

Intervals 1 + 1	Intervals 2 + 2	Intervals 3 + 3	Intervals 4 + 4	Intervals 5 + 5	Intervals 6 + 6
Intervals 1 + 2	Intervals 2 + 3	Intervals 3 + 4	Intervals 4 + 5	Intervals 5 + 6	
Intervals 1 + 3	Intervals 2 + 4	Intervals 3 + 5	Intervals 4 + 6	Intervals 5 + 7	
Intervals 1 + 4	Intervals 2 + 5	Intervals 3 + 6	Intervals 4 + 7		
Intervals 1 + 5	Intervals 2 + 6	Intervals 3 + 7	Intervals 4 + 8		
Intervals 1 + 6	Intervals 2 + 7	Intervals 3 + 8			
Intervals 1 + 7	Intervals 2 + 8	Intervals 3 + 9			
Intervals 1 + 8	Intervals 2 + 9				
Intervals 1 + 9	Intervals 2 + 10				
Intervals 1 + 10					
Intervals 1 + 11					

to Forte's approach, since he built genera with similar intuitive names such as 'chroma', 'dia', 'whole-tone', 'atonal', 'diminished' and 'augmented'.[41] However, rather than working from the twelve trichordal set classes *per se*, I would like to survey the wider field of ordered trichordal intervallic cells (which I will refer to as OTICs) or three-note motivic shapes as a source for investigation, in order to find additional ways of building generically beyond Hanson's projections and Eriksson's maxpoints. Since arithmetically there are 36 different ways of summing two integers of between 1 and 11 to totals of between 2 and 12, so there are 36 OTICs which can be formed from different combinations of two intervals of between interval 1 (the minor second) and interval 11 (the major seventh) within the twelve-note chromatic scale. Table 3 shows a listing of these 36 interval sums. The six interval sums equalling 12 (1 + 11, etc.) can be set aside since they produce dyadic pitch-class sets rather than trichordal ones, caused by the replication of a pitch at the octave. This leaves just 30 distinct OTICs. Each can be taken to represent transpositional and inversional forms of itself, but it has to remain ordered.[42] The 30 *a priori* OTICs are intimately connected to precisely 30 distinct combination (interval) cycles, that is, cycles that are generated from two overlapping forms of one particular interval cycle or from alternations of two different intervals.[43] Table 3 can be reformulated as Table 4 to show how each OTIC correlates one-to-one with the serially and continually repeating trichordal element within each of the combination cycles.[44]

Now that a specific relationship between OTICs and combination cycles has been established, further set classes of higher cardinality can be identified and placed within a larger generational context. These are shown in Table 5.[45] Column 1 of Table 5 shows each combination cycle and its own distinctive OTIC.[46] Column 2 sets out each pitch- and interval-class pattern, with the second (offset) interval cycle shown in bold. Column 3 gives the set-class name of each OTIC. It can be seen that there are three versions each of set classes 3–2 (0,1,3), 3–3 (0,1,4), 3–4 (0,1,5), 3–5 (0,1,6), 3–7 (0,2,5), 3–8 (0,2,6) and 3–11

Table 4 The relationship between the 30 OTICs and the 30 combination interval cycles

Combination interval 1 cycles (totalling 1 or 11, mod. 12)	Combination interval 2 cycles (totalling 2 or 10, mod. 12)	Combination interval 3 cycles (totalling 3 or 9, mod. 12)	Combination interval 4 cycles (totalling 4 or 8, mod. 12)	Combination interval 5 cycles (totalling 5 or 7, mod. 12)	Combination interval 6 cycles (totalling 6, mod. 12)
(1,10) (2,11)	(1,9) (3,11)	(1,8) (4,11)	(1,7) (5,11)	(1,6) (6,11)	(1,5) (7,11)
(2,9) (3,10)	(2,8) (4,10)	(2,7) (5,10)	(2,6) (6,10)	(2,5) (7,10)	(2,4) (8,10)
(3,8) (4,9)	(3,7) (5,9)	(3,6) (6,9)	(3,5) (7,9)	(3,4) (8,9)	(3,3) (9,9)
(4,7) (5,8)	(4,6) (6,8)	(4,5) (7,8)	(4,4) (8,8)	(1,4) (8,11)	
(5,6) (6,7)	(5,5) (7,7)	(1,2) (10,11)	(1,3) (9,11)	(2,3) (9,10)	
	(1,1) (11,11)		(2,2) (10,10)		

(0,3,7), two versions each of set classes 3–1 (0,1,2), 3–6 (0,2,4), 3–9 (0,2,7) and 3–10 (0,3,6), and just one version of set class 3–12 (0,4,8). This distribution reflects the trichords' varying degrees of symmetry and/or transpositional invariance. Column 4 shows the series of larger set classes of between four and twelve members generated serially by each of the combination cycles (and by each of the integral, overlapping OTICs). Looking at the second row – that is, the interval-1 combination cycle, (2,9) (3,10) – by way of example, we can see that the first generated tetrachord in column 4, 4–1 (0,1,2,3), unfolds serially from the first element in column 2 (0,2,11,1) and then again from the third element (11,1,10,0), and so on through the entire combination cycle, while the other generated tetrachord, 4–3 (0,1,3,4), similarly unfolds serially from the second element of the pattern (2,11,1,10) and then again from the fourth (1,10,0,9), and so on through the entire cycle. All of the larger generated sets in all 30 rows are derived in precisely the same way, from groups of contiguous elements of differing cardinality within the patterns of column 2. Each completed row in Table 5 therefore represents one basic OTIC and all of its larger generated sets, which together form an IG chain. Although Table 5 as a whole demonstrates 30 IG chains in all, a few are the same, replicating part or all of Hanson's projections (see again Fig. 2) and Eriksson's series of maxpoint set types (his ex. 2), and a few others are nearly identical.[47] What these patterns do show, however, is a wider range of correlated set classes than is found in Hanson's projections or Eriksson's maxpoint set types. This system of combination cycles therefore could potentially form the basis of a system of genera. The designated (intuitive) genus name for each type of correlated combination cycle (or each type of IG chain) can therefore be given as in column 5 of Table 5. In summary, the combination interval-1 cycles generate chromatic and bichromatic IG chains; the combination interval-2 cycles generate chromatic, whole-tone and diatonic IG chains; the combination interval-3 cycles generate purely octatonic IG chains; the combination interval-4 cycles generate hexatonic and whole-tone IG chains; the combination interval-5 cycles generate diatonic and bichromatic IG chains; and the combination interval-6 cycles generate bichromatic, octatonic and whole-tone

Table 5 The 30 combination interval cycles and OTICs and their generated set classes

Combination (c-) cycle and its OTIC	Pitch/interval-class pattern (offset cycle in bold)	Trichord set class	Larger generated set classes	System name
Interval 1 c-cycle (1,10) (2,11)	0,**1**,11,**0**,10,**11**,9,**10**,8,**9**,7,**8**,6, **7**,5,**6**,4,**5**,3,**4**,2,**3**,1,**2**,(0)	3–1 (0,1,2)	4–1, 5–1, 6–1, 7–1, 8–1, 9–1, 10–1, 11–1, 12–1	Chromatic
Interval 1 c-cycle (2,9) (3,10)	0,**2**,11,**1**,10,**0**,9,**11**,8,**10**,7, **9**,6,**8**,5,**7**,4,**6**,3,**5**,2,**4**,1,**3**,(0)	3–2 (0,1,3)	4–1/4–3, 5–1, 6–1, 7–1, 8–1, 9–1, 10–1, 11–1, 12–1	Chromatic
Interval 1 c-cycle (3,8) (4,9)	0,**3**,11,**2**,10,**1**,9,**0**,8,**11**,7,**10**, 6,**9**,5,**8**,4,**7**,3,**6**,2,**5**,1,**4**,(0)	3–3 (0,1,4)	4–3/4–7, 5–3, 6–1/6–Z4, 7–1, 8–1, 9–1, 10–1, 11–1, 12–1	Chromatic
Interval 1 c-cycle (4,7) (5,8)	0,**4**,11,**3**,10,**2**,9,**1**,8,**0**,7,**11**, 6,**10**,5,**9**,4,**8**,3,**7**,2,**6**,1,**5**,(0)	3–4 (0,1,5)	4–7 (chr)/4–8 (bichr), 5–6, 6Z4 (chr)/6–Z6 (bichr), 7–5, 8–1(chr)/8–6 (bichr), 9–1, 10–1, 11–1, 12–1	Chromatic/ bichromatic
Interval 1 c-cycle (5,6) (6,7)	0,**5**,11,**4**,10,**3**,9,**2**,8,**1**,7,**0**,6, **11**,5,**10**,4,**9**,3,**8**,2,**7**,1,**6**,(0)	3–5 (0,1,6)	4–8/4–9, 5–7, 6–Z6/6–7, 7–7, 8–6/8–9, 9–5, 10–1/10–6, 11–1, 12–1	Bichromatic
Interval 2 c-cycle (1,1) (11,11)	0,**1**,2,**3**,4,**5**,6,**7**,8,**9**,10,**11**,(0)	3–1 (0,1,2)	4–1, 5–1, 6–1, 7–1, 8–1, 9–1, 10–1, 11–1, 12–1	Chromatic
Interval-2 c-cycle (1,9) (3,11)	0,**1**,10,**11**,8,**9**,6,**7**,4,**5**,2,**3**,(0)	3–2 (0,1,3)	4–1/4–10, 5–2, 6–1/6–8, 7–2, 8–1/8–10, 9–2, 10–1/10–3, 11–1, 12–1	Chromatic
Interval 2 c-cycle (2,8) (4,10)	0,**2**,10,**0**,8,**10**,6,**8**,4,**6**,2,**4**,(0)	3–6 (0,2,4)	4–21, 5–33, 6–35	Whole-tone
Interval 2 c-cycle, (3,7) (5,9)	0,**3**,10,**1**,8,**11**,6,**9**,4,**7**,2,**5**,(0)	3–7 (0,2,5)	4–10/4–23, 5–23, 6–8/6–32, 7–23, 8–10/8–23, 9–7, 10–3/10–5, 11–1, 12–1	Diatonic
Interval 2 c-cycle (4,6) (6,8)	0,**4**,10,**2**,8,**0**,6,**10**,4,**8**,2,**6**,(0)	3–8 (0,2,6)	4–21/4–25, 5–33, 6–35	Whole-tone
Interval 2 c-cycle (5,5) (7,7)	0,**5**,10,**3**,8,**1**,6,**11**,4,**9**,2,**7**,(0)	3–9 (0,2,7)	4–23, 5–35, 6–32, 7–35, 8–23, 9–9, 10–5, 11–1, 12–1	Diatonic
Interval 3 c-cycle (1,2) (10,11)	0,**1**,3,**4**,6,**7**,9,**10**,(0)	3–2 (0,1,3)	4–3/4–10, 5–10, 6–Z13/6–Z23, 7–31, 8–28	Octatonic
Interval 3 c-cycle (1,8) (4,11)	0,**1**,9,**10**,6,**7**,3,**4**,(0)	3–3 (0,1,4)	4–3/4–17, 5–16,6–Z13/6–Z49, 7–31, 8–28	Octatonic
Interval 3 c-cycle (2,7) (5,10)	0,**2**,9,**11**,6,**8**,3,**5**,(0)	3–7 (0,2,5)	4–10/4–26, 5–25, 6–Z23/6–Z50, 7–31, 8–28	Octatonic
Interval 3 c-cycle (3,6) (6,9)	0,**3**,9,**0**,6,**9**,3,**6**,(0)	3–10 (0,3,6)	4–28	Octatonic
Interval 3 c-cycle (4,5) (7,8)	0,**4**,9,**1**,6,**10**,3,**7**,(0)	3–11 (0,3,7)	4–17/4–26, 5–32, 6–Z49/6–Z50, 7–31, 8–28	Octatonic

Table 5 Continued

Combination (c-) cycle and its OTIC	Pitch/interval-class pattern (offset cycle in bold)	Trichord set class	Larger generated set classes	System name
Interval 4 c-cycle (1,3) (9,11)	0,**1**,4,**5**,8,**9**,(0)	3–3 (0,1,4)	4–7/4–17, 5–21, 6–20	Hexatonic
Interval 4 c-cycle (1,7) (5,11)	0,**1**,8,**9**,4,**5**,(0)	3–4 (0,1,5)	4–7/4–20, 5–21, 6–20	Hexatonic
Interval 4 c-cycle (2,2) (10,10)	0,**2**,4,**6**,8,**10**,(0)	3–6 (0,2,4)	4–21, 5–33, 6–35	Whole-tone
Interval 4 c-cycle (2,6) (6,10)	0,**2**,8,**10**,4,**6**,(0)	3–8 (0,2,6)	4–21/4–25, 5–33, 6–35	Whole-tone
Interval 4 c–cycle (3,5) (7,9)	0,**3**,8,**11**,4,7,(0)	3–11 (0,3,7)	4–17/4–20, 5–21, 6–20	Hexatonic
Interval 4 c-cycle (4,4) (8,8)	0,**4**,8,**0**,4,**8**, (0)	3–12 (0,4,8)		Hexatonic or whole-tone
Interval 5 c-cycle (1,4) (8,11)	0,**1**,5,**6**,10,**11**,3,**4**,8,**9**,1,**2**,6, **7**,11,**0**,4,**5**,9,**10**,2,**3**,7,**8**,(0)	3–4 (0,1,5)	4–8 (bichr)/4–20 (dia), 5–20, 6–Z3 (bichr)/ 6–Z26 (dia), 7–14, 8–6 (bichr)/8–23 (dia), 9–9, 10–5, 11–1, 12–1	Bichromatic/ diatonic
Interval 5 c-cycle (1,6) (6,11)	0,**1**,7,**8**,2,**3**,9,**10**,4,**5**,11,**0**,6, **7**,1,**2**,8,**9**,3,**4**,10,**11**,5,**6**,(0)	3–5 (0,1,6)	4–8/4–9, 5–7, 6–7/6–Z38, 7–7, 8–6/8–9, 9–5, 10–5/10–6, 11–1, 12–1	Bichromatic
Interval 5 c-cycle (2,3) (9,10)	0,**2**,5,7,10,**0**,3,**5**,8,**10**,1,**3**,6, **8**,11,**1**,4,**6**,9,**11**,2,**4**,7,**9**,(0)	3–7 (0,2,5)	4–23/4–26, 5–35, 6–32, 7–35, 8–23, 9–9, 10–5, 11–1, 12–1	Diatonic
Interval 5 c-cycle (2,5) (7,10)	0,**2**,7,**9**,2,**4**,9,**11**,4,**6**,11,**1**,6, **8**,1,**3**,8,**10**,3,**5**,10,**0**,5,7,(0)	3–9 (0,2,7)	4–23, 5–35, 6–32, 7–35, 8–23, 9–9, 10–5, 11–1, 12–1	Diatonic
Interval 5 c-cycle (3,4) (8,9)	0,**3**,7,**10**,2,**5**,9,**0**,4,7,11,**2**,6, **9**,1,**4**,8,**11**,3,**6**,10,**1**,5,**8**,(0)	3–11 (0,3,7)	4–20/4–26, 5–27, 6–Z26/6–32, 7–35, 8–23, 9–9, 10–5, 11–1, 12–1	Diatonic
Interval 6 c-cycle (1,5) (7,11)	0,**1**,6,7,(0)	3–5 (0,1,6)	4–9	Bichromatic or octatonic
Interval 6 c-cycle (2,4) (8,10)	0,**2**,6,**8**,(0)	3–8 (0,2,6)	4–25	Bichromatic, octatonic or whole-tone
Interval 6 c-cycle (3,3) (9,9)	0,**3**,6,**9**,(0)	3–10 (0,3,6)	4–28	Octatonic

IG chains. This all correlates to those positive characteristic features identified for each prototypical hexachord in Table 2. We can now begin to see a connection between the bichromatic area and the chromatic and diatonic ones, and to appreciate the multiple generative roles played by each of the interval classes.

Thus far, although this undertaking has tentatively assigned all six dyadic and twelve trichordal set classes to one or more of the intuitive genera, relatively few of the 29 tetrachords, 38 pentachords and 50 hexachords have been incorporated; clearly a fresh and far more rigorous approach will be required in order to meet the need for set-class completeness and, beyond this, to provide the possibility of worthwhile intergeneric intersection.[48] We can begin this task by instigating the creation of six hexachord 'families' to include all 50 of the hexachordal set classes, so that each of them not only can provide the representative hexachordal membership and backbone for each of the six intuitive genera, but also can make provision, beyond this completeness, for some degree of intergeneric overlap.

As a first step towards this goal, it will be helpful to look at one particular aspect of the generated set classes in column 4 of Table 5, where we can see that the cardinal 6 component of some of the IG chains extends beyond the six designated prototypical hexachords to other hexachords via a common subset or superset of adjacent cardinality.[49] This inclusional aspect of similarity, that set classes within any genus could have an 'almost-the-same-as' status through a 'subset-of-prototype-plus' or 'superset-of-prototype-minus' relational characteristic, is a principle which can now be extended in order to to include all of the hexachordal set classes. This approach will be followed throughout the ensuing creative process in order to embrace those hexachords with the closest affinity to each generic prototype. As a useful preliminary to this course of action, we can look at the two principal combination interval-cycle patterns allocated to the bichromatic system by Table 5: the interval-1 (5,6) (6,7) and the interval-5 (1,6) (6,11). We can see that the hexachords present in these patterns, 6–7 (0,1,2,6,7,8), 6–Z6 (0,1,2,5,6,7) and 6–Z38 (0,1,2,3,7,8), all contain set class 5–7 (0,1,2,6,7). Thus we can say that, in these contexts, set class 5–7 conserves the 'bichromatic' quality of each and serves to underpin two equivalent modifications to Hanson's basic 6–7-centred projection (see again Fig. 2): 6–Z6 and 6–Z38 can be viewed either as minimal modifications of the bichromatic prototype, 6–7, or as maximally similar derivatives of it. Now, it transpires that all 44 of the non-prototypical hexachords, without exception, can be categorised in a similar way, such that their prime forms are minimal modifications or maximally similar derivatives of the prime forms of one or more of the six hexachordal prototypes.[50] It must be pointed out at this juncture that, although these modifications are minimal in terms of common notes, the actual intervallic displacement of one element is not necessarily minimal in voice-leading terms: for the purposes of this process, any size of intervallic displacement, from the semitone to the tritone, is of equal validity.[51]

The resultant hexachord families of 'near relatives' are set out in Table 6. In all cases, family membership is ensured through a limited number (between one and three) of common 'foundational' pentachords, which are themselves the only pentachordal subsets of the progenitor hexachords and therefore underpin their systems.[52] Hexachords exclusive to just one family are shown in bold in Table 6, foundational pentachords are placed to the right of each hexachord and the prime-form element of each hexachord that is foreign to its foundational pentachord is shown in bold. These families correlate largely with the cardinal 6 aspect of Eriksson's regions and correspond exactly with Tenkanen's 'cornerstone hexachords and their nearest relatives', although neither of these sets of collections demonstrates any aspect of generic intersection other than diatonic-with-chromatic, seriously limiting the possibility that intergeneric affinity across the other cardinalities might be induced from them.[53] At this point in the process we can begin to meet some of Quinn's desiderata: the familial hexachord structure shown in Table 6 satisfies elements of key criteria 2, 5 and 6, that 'quality space' should be determined by similarity relations, that sets belonging to a single genus should have intrageneric affinities and that equivalence classes are an integral part of (what constitutes) 'quality' between set classes.

It was noted earlier that computationally generated set-class similarity measures have typically derived either from subset/superset inclusional comparisons or from interval-class vector rankings, but that comparable results can often emerge from both procedures. Unsurprisingly, it transpires that an interval-class vector analysis of the 50 hexachords will produce much the same spread within families as does the subset/superset inclusional matching of Table 6. We can start to perform this analysis by observing that incremental building from the six interval classes, as illustrated in the Hanson IG chains of Fig. 2, will produce patterns of interval-class vector growth, as shown in Table 7, which correlate exactly with the interval-class vector patterns exemplified by the prototype hexachords in Table 2. Looking next at the interval-class vectors of all 50 hexachords, as allocated in the families of Table 6, we can see that there is still a tendency for them to follow the characteristic aspects of the patterns in Table 7, although not as abundantly. To illustrate, within the hexatonic family the ic 2 and ic 6 content for the maximally similar hexachords is now slightly higher than 6–20's zero (because slightly less completely and uniquely hexatonic), while the content of the other five ics, including the defining ic 4, is slightly lower (also because slightly less completely and uniquely hexatonic). All six constituents of these seven hexachords' interval-class vectors reflect their slightly changed 'hexatonic' status. Thus, it can be asserted that, although equivalence or similarity between the members of any hexachord family in Table 6 has been based on inclusion, there is a degree of equivalence or similarity based on interval-class vectors as well. Beyond this, Eriksson's ex. 4 has shown that similar interval-class vector trends, as represented in his mm vector maxpoint groups, can be discerned to varying degrees in the generality of set classes. Meanwhile, his ex. 6 has shown what ought to epitomise each genus type in terms of notably high or low

Table 6 Six hexachord families of equivalence/similarity, generated from hexachordal prototypes and based on common foundational pentachords

	Hexatonic			Bichromatic			Octatonic			Whole-Tone			Diatonic			Chromatic	
6–20	**(0,1,4,5,8,9)**		**6–7**	**(0,1,2,6,7,8)**		**6–30**	**(0,1,3,6,7,9)**		**6–35**	**(0,2,4,6,8,10)**		**6–32**	**(0,2,4,5,7,9)**		**6–1**	**(0,1,2,3,4,5)**	
6–Z19	(0,**1**,3,4,7,8)	5–21	**6–Z6**	(0,1,2,**5**,6,7)	5–7	6–Z12	(0,1,**2**,4,6,7)	5–19	6–21	(0,2,**3**,4,6,8)	5–33	**6–Z24**	(**0**,1,3,4,6,8)	5–23	**6–Z3**	(**0**,1,2,3,5,**6**)	5–2/5–3
6–Z44	(**0**,1,2,5,6,9)	5–21	**6–Z38**	(0,1,2,**3**,7,8)	5–7	6–Z41	(0,**1**,2,3,6,8)	5–28	6–22	(0,**1**,2,4,6,8)	5–33	**6–Z46**	(**0**,1,2,4,6,9)	5–27	**6–Z36**	(0,1,2,3,4,**7**)	5–1
									6–34	(**0**,1,3,5,7,9)	5–33						
6–14	(0,1,**3**,4,5,8)	5–21	6–Z12	(0,1,2,**4**,6,7)	5–7	**6–Z13**	(0,1,3,**4**,6,7)	5–19				**6–Z25**	(**0**,1,3,5,**6**,8)	5–23/5–27	**6–Z4**	(0,1,2,4,5,**6**)	5–3
6–15	(0,1,**2**,4,5,8)	5–21	6–Z41	(0,1,2,**3**,6,8)	5–15	**6–Z42**	(0,1,**2**,3,6,9)	5–31				**6–Z47**	(0,**1**,2,4,7,9)	5–35	**6–Z37**	(0,1,2,3,4,**8**)	5–1
6–16	(0,1,4,5,**6**,8)	5–21															
6–31	(0,1,**3**,5,8,9)	5–21	6–Z17	(0,1,2,**4**,7,8)	5–7	6–Z17	(**0**,1,2,4,7,8)	5–19				**6–Z26**	(0,**1**,3,5,7,8)	5–27	**6–Z10**	(0,1,3,4,5,**7**)	5–3
			6–Z43	(0,1,2,**5**,6,8)	5–15	6–Z43	(0,**1**,2,5,6,8)	5–28				**6–Z48**	(0,**1**,2,5,7,9)	5–35	**6–Z39**	(0,2,3,4,5,**8**)	5–2
			6–5	(0,1,2,**3**,6,7)	5–7	**6–Z23**	(0,2,3,**5**,6,8)	5–28				6–Z11	(0,**1**,2,4,5,7)	5–23	6–Z11	(0,1,2,4,5,**7**)	5–3
			6–18	(0,1,2,**5**,7,8)	5–7	**6–Z45**	(0,2,3,**4**,6,9)	5–31				6–Z40	(0,1,**2**,3,5,8)	5–27	6–Z40	(0,1,2,3,5,**8**)	5–2
			6–22	(0,1,2,**4**,6,8)	5–15												
						6–Z28	(0,1,3,**5**,6,9)	5–31				6–8	(0,2,**3**,4,5,7)	5–23	**6–2**	(**0**,1,2,3,4,**6**)	5–1/5–2
						6–Z49	(0,1,3,**4**,7,9)	5–28				6–9	(0,**1**,2,3,5,7)	5–23	6–8	(**0**,2,3,4,5,7)	5–2
												6–14	(0,1,3,**4**,5,8)	5–27	6–9	(0,1,2,3,5,**7**)	5–2
						6–Z29	(0,1,3,6,**8**,9)	5–31				6–31	(0,1,3,5,8,**9**)	5–27	6–14	(0,1,3,4,5,**8**)	5–3
						6–Z50	(0,1,**4**,6,7,9)	5–19				**6–33**	(0,2,**3**,5,7,**9**)	5–23/5–35	6–15	(0,1,2,4,5,**8**)	5–3
						6–5	(0,1,**2**,3,6,7)	5–19									
						6–18	(**0**,1,2,5,7,8)	5–19									
						6–21	(0,2,3,**4**,6,8)	5–28									
						6–27	(0,**1**,3,**4**,6,9)	5–31									
						6–34	(0,1,3,**5**,7,9)	5–28									

Note: Genus prototypes are placed at the top of each column. The prime form element of each hexachord foreign to its foundational pentachord is shown in bold; two prime form numerals in bold indicate alternative foreign elements.

Table 7 Interval-class vector growth in the Hanson interval-class projections

ic 4 build	2–4 [000100]	3–12 [000300]	4–19 [101310]	5–21 [202420]	6–20 [303630]	7–21 [424641]	8–19 [545752]	9–12 [666963]	10–4 [888984]	Increasing ic 4 first/most	Increasing ics 2 and 6 last/least
ic 3 build	2–3 [001000]	3–10 [002001]	4–28 [004002]	5–31 [114112]	6–27 [225222] 6–30 [224223]	7–31 [336333]	8–28 [448444]	9–10 [668664]	10–3 [889884]	Increasing ics 3 and 6 first/most	Increasing ics 2, 3, 4 and 5 later/less, and evenly
ic 2 build	2–2 [010000]	3–6 [020100]	4–21 [030201]	5–33 [040402]	6–35 [060603]	7–33 [262623]	8–21 [474643]	9–6 [686763]	10–2 [898884]	Increasing ics 2 4 and 6 first/most	Increasing ics 1, 3 and 5 last/least
ic 5 build	2–5 [000010]	3–9 [010020]	4–23 [021030]	5–35 [032140]	6–32 [143250]	7–35 [254361]	8–23 [465472]	9–9 [676683]	10–5 [888894]	Increasing in the order ic 5, ic 2, ic 3, ic 4, ic 1, ic 6	
ic 1 build	2–1 [100000]	3–1 [210000]	4–1 [321000]	5–1 [432100]	6–1 [543210]	7–1 [654321]	8–1 [765442]	9–1 [876663]	10–1 [988884]	Increasing in the order ic 1, ic 2, ic 3, ic 4, ic 5, ic 6	
ic 6 build	2–6 [000001]	3–5 [100011]	4–9 [200022]	5–7 [310132]	6–7 [420243]	7–7 [532353]	8–9 [644464]	9–5 [766674]	10–6 [888885]	Increasing ics 1, 5 and 6 first/most	Increasing ic 3 last/least

Note: In each interval-class vector, interval classes increasing first/most are shown in the largest font and interval classes increasing last/least are shown in the smallest font (ic 6 values should effectively be doubled).

ic counts, and what distinguishes one genus from another.[54] These two examples go some distance towards satisfying key criteria 5 and 8: that set classes belonging to a single genus 'would lie near one another' and that intrageneric affinities are arranged around 'privileged, highly symmetric' prototypes which are 'quite distant from one another'. Eriksson's ex. 6 closely resembles Buchler's corresponding i-set-based chart (see Buchler's fig. 5).[55] Eriksson's ex. 7, which illustrates his mm vector-induced regions, has widened the range of membership of his genera to include all set classes, with the hexachord component of each of his regions correlating closely to the hexachord families of Table 6. Eriksson's array also shows a fairly close relationship to the Monte Carlo–type and cluster analysis–induced regions collated by Quinn.[56]

Although Eriksson and Quinn have achieved completion of the set-class universe through interval-class vector analysis, their regions do not provide sufficient incidence of genus overlap; only Forte's genus system has done so. It seems that, in the majority of cases, Eriksson's examination of interval-class vector characteristics and Quinn's analyses of interval-class similarity measures have assigned each set class to just one genus. But while there is surely a need to extend and refine membership of any genera beyond that provided by Hanson, Eriksson and Quinn, there is also just as strong a need to limit this membership in order to avoid the profligacy engendered by Forte. Any further family-based expansion similar to that found in Table 6 would unfortunately result in overlarge genus membership in the other cardinalities, creating some of the problems already displayed by Forte's system. Therefore, in order to expand membership of my genera to include a sufficient but balanced degree of intergeneric affinity (key criterion 5), I propose that the inclusional road of subset/superset IG chaining should now be explored further. Before embarking on this new direction, however, a major stumbling block has to be surmounted, namely the capacity for subset embedding to escalate as the difference between cardinalities grows, resulting in an accumulation of myriad inclusional relationships across the cardinalities.[57]

While IG chaining based on prime-form similarity might provide the best rationale for the evolution of a lucid and uncluttered system of genera, clearly some form of constraint will be needed to keep membership and intergeneric intersection to manageable proportions. It will therefore be desirable to create a composite form of IG chaining that incorporates bonded paired or grouped set classes at each cardinal stage as a core constructional principle, with subset(s)/superset(s) of the genus prototype, or hexachord(s) from the genus hexachord family, at its limits. This method of bonding will focus the intrinsic construction of the intermediate cardinal stages within required, genera-founded limits. The bonded pattern of applied inclusivity between set classes of adjacent cardinalities might also be extended beyond cardinal 6 to the point where it reaches its natural conclusion. This method of 'bonded IG' (henceforth referred to as 'BIG') chaining can be amalgamated with the hexachord families of Table 6 to create a properly complete and consistent system of classification, which will in turn form the basis for a new, comprehensive pitch-class set system of genera. Given the empirical desire to categorise these BIG chains according to the proposed intuitive genera and to the hexachord families of Table 6, and given the equally candid desire to create genera of manageable proportions with watertight conditions for membership, the primary criteria for the creation of any BIG chain and its ultimate allocation to a generic 'type' will have to be defined precisely and then strictly adhered to. As a logical consequence of the argument so far, these criteria would be:

1. that any hexachord content complies to the generic hexachord family;
2. that initial set class(es) of lowest cardinality are either subset(s) of the defining genus progenitor hexachord or, if the BIG chain begins with a hexachord, member(s) of the generic hexachord family;
3. that final set class(es) of highest cardinality are either superset(s) of the defining genus progenitor hexachord or, if the BIG chain terminates with a hexachord, member(s) of the generic hexachord family;
4. that intermediate pairs of set classes of the same cardinality adhere strictly to an all-inclusive network of set-to-set interlacing and binding pathways, where *both* members of the pair are subsets or supersets of the set class(es) of an adjacent cardinality, as illustrated in Fig. 3; and
5. that BIG chains display at least one of the bonded criss-cross patterns of inclusivity shown in Fig. 3.[58]

Following these procedures, all validly created BIG chains can be securely categorised and listed under the intuitive genus names that head Tables A1–A6 (see the Appendix).[59] The only exception to the above criteria that needs to be made concerns those fundamental Hanson/Eriksson/Buchler IG (non-bonded) chains for each intuitive genus which have simpler one-to-one pathways, but which must nevertheless be allowed into the appropriate categories of Tables A1–A6, either because they have proved initiatory to the whole ensuing processes of generation (such as 6–20 (0,1,4,5,8,9), the hexatonic prototype) or because they act as host to certain singular set classes of high invariance that could not figure as elements within the BIG form of chaining. These singular IG chains are labelled either 'Hanson/Eriksson IG chain' or 'Hanson/Eriksson/Buchler IG chain' in Tables A1–A6.[60] Every set class present under the intuitive genus names of Tables A1–A6 can now be transferred to the corresponding six genera, as classified in Table 8.[61] The *modus operandi* displayed in the creation of Tables A1–A6 and Table 8 conforms to key criterion 3 in that it presents

Fig. 3 An example of a BIG chain's bonded pattern of subset/superset inclusivity

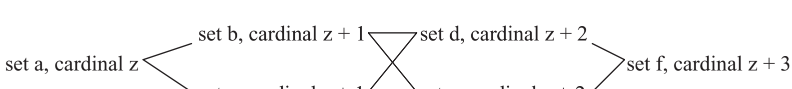


Table 8 A hexachord-derived pitch-class set system of genera

(a) Hexatonic genus

2–1+ (0,1)	3–1+ (0,1,2)	4–2+ (0,1,2,4)	5–3 (0,1,2,4,5)	6–14e (0,1,3,4,5,8)
2–3+ (0,3)	**3–3**e **(0,1,4)**	4–4 + (0,1,2,5)	5–6+ (0,1,2,5,6)	6–15e (0,1,2,4,5,8)
{2–4e+ (0,4)}	**3–4e+ (0,1,5)**	4–5 + (0,1,2,6)	5–11e (0,2,3,4,7)	6–16e⋆ (0,1,4,5,6,8)
2–5+ (0,5)	3–9+ (0,2,7)	**4–7**e **(0,1,4,5)**	5–13 (0,1,2,4,8)	6–Z19e⋆ (0,1,3,4,7,8)
	3–10+ (0,3,6)	4–12+ (0,2,3,6)	5–16 (0,1,3,4,7)	**6–20e⋆ (0,1,4,5,8,9)**
	3–11e **(0,3,7)**	4–14+ (0,2,3,7)	5–Z17e⋆ (0,1,3,4,8)	6–31e (0,1,3,5,8,9)
	3–12e **(0,4,8)**	4–16+ (0,1,5,7)	5–Z18e+ (0,1,4,5,7)	6–Z44e⋆ (0,1,2,5,6,9)
		4–17e **(0,3,4,7)**	5–20+ (0,1,3,7,8)	
		4–18+ (0,1,4,7)	**5–21e⋆ (0,1,4,5,8)**	
		4–19e **(0,1,4,8)**	5–22e⋆ (0,1,4,7,8)	
		4–20e **(0,1,5,8)**	5–26 (0,2,4,5,8)	
		4–22+ (0,2,4,7)	5–27 (0,1,3,5,8)	
		4–24 (0,2,4,8)	5–30 (0,1,4,6,8)	
		4–27+ (0,2,5,8)	5–32 (0,1,4,6,9)	
			5–Z37e⋆ (0,3,4,5,8)	
			5–Z38e+ (0,1,2,5,8)	

(b) Bichromatic genus

2–1+ (0,1)	**3–1+ (0,1,2)**	4–1 (0,1,2,3)	5–4e (0,1,2,3,6)	6–5e (0,1,2,3,6,7)
{**2–2+ (0.2)**}	**3–4+ (0,1,5)**	4–2+ (0,1,2,4)	5–5e (0,1,2,3,7)	6–Z6e⋆ (0,1,2,5,6,7)
{**2–4+ (0,4)**}	**3–5**e **(0,1,6)**	4–4+ (0,1,2,5)	5–6e+ (0,1,2,5,6)	**6–7e⋆ (0,1,2,6,7,8)**
2–5+ (0,5)	**3–8 (0,2,6)**	**4–5e+ (0,1,2,6)**	**5–7**e **(0,1,2,6,7)**	6–Z12e (0,1,2,4,6,7)
2–6e **(0,6)**	**3–9+ (0,2,7)**	**4–6e⋆ (0,1,2,7)**	5–9+ (0,1,2,4,6)	6–Z17e (0,1,2,4,7,8)
		4–8e⋆ (0,1,5,6)	{5–Z12e (0,1,3,5,6)}	6–18e (0,1,2,5,7,8)
		4–9e **(0,1 6 7)**	5–13 (0,1,2,4,8)	6–22 (0,1,2,4,6,8)
		4–13+ (0,1,3,6)	5–14e (0,1,2,5,7)	6–Z38e⋆ (0,1,2,3,7,8)
		4–14+ (0,2,3,7)	**5–15**e **(0,1,2,6,8)**	6–Z41e (0,1,2,3,6,8)
		4–16e+ (0,1,5,7)	5–Z18e+ (0,1,4,5,7)	6–Z43e (0,1,2,5,6,8)
		4–18+ (0,1,4,7)	5–19e (0,1,3,6,7)	
		4–22+ (0,2,4,7)	5–20e+ (0,1,3,7,8)	
		4–23 (0,2,5,7)	5–24+ (0,1,3,5,7)	
		4–25 (0,2,6,8)	5–28 (0,2,3,6,8)	
			5–29e (0,1,3,6,8)	
			5–30 (0,1,4,6,8)	
			5–Z36e+ (0,1,2,4,7)	
			5–Z38e+ (0,1,2,5,8)	

(c) Octatonic genus

2–1+ (0,1)	3–1+ (0,1,2)	4–3e† (0,1,3,4)	5–4 (0,1,2,3,6)	6–5 (0,1,2,3,6,7)
2–2+ (0,2)	**3–2 (0,1,3)**	4–4+ (0,1,2,5)	5–6+ (0,1,2,5,6)	6–Z12 (0,1,2,4,6,7)
2–3e+ (0,3)	**3–3 (0,1,4)**	4–5+ (0,1,2,6)	5–7 (0,1,2,6,7)	6–Z13e⋆ (0,1,3,4,6,7)
2–4+ (0,4)	3–4+ (0,1,5)	**4–9 (0,1,6,7)**	5–10e† (0,1,3,4,6)	6–Z17 (0,1,2,4,7,8)
2–5+ (0,5)	**3–5 (0,1,6)**	4–10e† (0,2,3,5)	{5–Z12e (0,1,2,3,5,6)}	6–18 (0,1,2,5,7,8)
2–6e (0,6)	**3–7 (0,2,5)**	**4–12e+ (0,2,3,6)**	5–15 (0,1,2,6,8)	6–21 (0,2,3,4,6,8)
	3–8 (0,2,6)	**4–13e+ (0,1,3,5)**	5–16e† (0,1,3,4,7)	6–Z23e⋆ (0,2,3,5,6,8)
	3–9+ (0,2,7)	4–14+ (0,2,3,7)	5–Z18e+ (0,1,4,5,7)	6–27e⋆ (0,1,3,4,6,9)
	3–10e+ (0,3,6)	**4–Z15e⋆ (0,1,4,6)**	**5–19e (0,1,3,6,7)**	6–Z28e⋆ (0,1,3,5,6,9)
	3–11 (0,3,7)	4–16+ (0,1,5,7)	5–20+ (0,1,3,7,8)	6–Z29e⋆ (0,1,3,6,8,9)
		4–17† (0,3,4,7)	5–25e† (0,2,3,5,8)	**6–30e⋆ (0,1,3,6,7,9)**
		4–18e+ (0,1,4,7)	5–26 (0,2,4,5,8)	6–34 (0,1,3,5,7,9)
		4–25e **(0,2,6,8)**	**5–28**e **(0,2,3,6,8)**	6–Z41 (0,1,2,3,6,8)
		4–26† (0,3,5,8)	5–29 (0,1,3,6,8)	6–Z42e⋆ (0,1,2,3,6,9)
		4–27+ (0,2,5,8)	**5–31e⋆ (0,1,3,6,9)**	6–Z43 (0,1,2,5,6,8)
		4–28e⋆ (0,3,6,9)	5–32e† (0,1,4,6,9)	6–Z45e⋆ (0,2,3,4,6,9)
		4–Z29e⋆ (0,1,3,7)	5–Z36e+ (0,1,2,4,7)	6–Z49e⋆ (0,1,3,4,7,9)
			5–Z38e+ (0,1,2,5,8)	6–Z50e⋆ (0,1,4,6,7,9)

Table 8 Continued

(d) Whole-Tone genus

2–2e+ (0,2)	**3–6e (0,2,4)**	4–2+ (0,1,2,4)	5–8e (0,2,3,4,6)	6–21e (0,2,3,4,6,8)
2–4e+ (0,4)	**3–8e (0,2,6)**	4–11 (0,1,3,5)	5–9e+ (0,1,2,4,6)	6–22e (0,1,2,4,6,8)
2–6e (0,6)	**3–12e (0,4,8)**	4–12+ (0,2,3,6)	5–13e (0,1,2,4,8)	6–34e (0,1,3,5,7,9)
		4–19 (0,1,4,8)	5–24e+ (0,1,3,5,7)	**6–35e⋆ (0,2,4,6,8,10)**
		4–21e (0,2,4,6)	5–26e (0,2,4,5,8)	
		4–22+ (0,2,4,7)	5–28e (0,2,3,6,8)	
		4–24e (0,2,4,8)	5–30e (0,1,4,6,8)	
		4–25e (0,2,6,8)	**5–33e⋆ (0,2,4,6,8)**	
		4–27+ (0,2,5,8)	5–34e (0,2,4,6,9)	

(e) Diatonic genus

2–1+ (0,1)	**3–2 (0,1,3)**	4–2+ (0,1,2,4)	5–9+ (0,1,2,4,6)	6–8e (0,2,3,4,5,7)
2–2e+ (0,2)	**3–4+ (0,1,5)**	**4–10e (0,2,3,5)**	5–10 (0,1,3,4,6)	6–9e (0,1,2,3,5,7)
2–3+ (0,3)	**3–6 (0,2,4)**	**4–11e (0,1,3,5)**	5–11e (0,2,3,4,7)	6–Z11e (0,1,2,4,5,7)
2–4+ (0,4)	**3–7e (0,2,5)**	4–13+ (0,1,3,6)	{5–Z12 (0,1,3,5,6)}	6–14 (0,1,3,4,5,8)
2–5e+ (0,5)	**3–9e+ (0,2,7)**	**4–14e+ (0,2,3,7)**	5–14e (0,1,2,5,7)	6–Z24e⋆ (0,1,3,4,6,8)
	3–10+ (0,3,6)	4–16+ (0,1,5,7)	5–20e+ (0,1,3,7,8)	6–Z25e⋆ (0,1,3,5,6,8)
	3–11 (0,3,7)	4–18+ (0,1,4,7)	**5–23e⋆ (0,2,3,5,7)**	6–Z26e⋆ (0,1,3,5,7,8)
		4–20 (0,1,5,8)	5–24+ (0,1,3,5,7)	6–31 (0,1,3,5,8,9)
		4–21 (0,2,4,6)	5–25 (0,2,3,5,8)	**6–32e⋆ (0,2,4,5,7,9)**
		4–22e+ (0,2,4,7)	**5–27e (0,1,3,5,8)**	6–33e⋆ (0,2,3,5,7,9)
		4–23e (0,2,5,7)	5–29e (0,1,3,6,8)	6–Z40e (0,1,2,3,5,8)
		4–26e (0,3,5,8)	5–32 (0,1,4,6,9)	6–Z46e⋆ (0,1,2,4,6,9)
		4–27+ (0,2,5,8)	5–34 (0,2,4,6,9)	6–Z47e⋆ (0,1,2,4,7,9)
			5–35e⋆ (0,2,4,7,9)	6–Z48e⋆ (0,1,2,5,7,9)
			5–Z36+ (0,1,2,4,7)	
			5–Z38 + (0,1,2,5,8)	

(f) Chromatic genus

2–1e+ (0,1)	**3–1e+ (0,1,2)**	**4–1e (0,1,2,3)**	**5–1e⋆ (0,1,2,3,4)**	**6–1e⋆ (0,1,2,3,4,5)**
2–2e+ (0,2)	**3–2e (0,1,3)**	**4–2e+ (0,1,2,4)**	**5–2e⋆ (0,1,2,3,5)**	6–2e⋆ (0,1,2,3,4,6)
2–3+ (0,3)	**3–3 (0,1,4)**	**4–3e (0,1,3,4)**	**5–3e (0,1,2,4,5)**	6–Z3e⋆ (0,1,2,3,5,6)
2–4+ (0,4)	**3–4+ (0,1,5)**	**4–4e+ (0,1,2,5)**	5–4e (0,1,2,3,6)	6–Z4e⋆ (0,1,2,4,5,6)
2–5+ (0,5)	**3–6 (0,2,4)**	4–5+ (0,1,2,6)	5–5e (0,1,2,3,7)	6–8e (0,2,3,4,5,7)
	3–7 (0,2,5)	**4–7 (0,1,4,5)**	5–6+ (0,1,2,5,6)	6–9e (0,1,2,3,5,7)
	3–10+ (0,3,6)	**4–10e (0,2,3,5)**	5–8 (0,2,3,4,6)	6–Z10e⋆ (0,1,3,4,5,7)
		4–11e (0,1,3,5)	5–9+ (0,1,2,4,6)	6–Z11e (0,1,2,4,5,7)
		4–12+ (0,2,3,6)	5–10 (0,1,3,4,6)	6–14 (0,1,3,4,5,8)
		4–13+ (0,1,3,6)	5–11e (0,2,3,4,7)	6–15 (0,1,2,4,5,8)
		4–18+ (0,1,4,7)	{5–Z12 (0,1,3,5,6)}	6–Z36e⋆ (0,1,2,3,4,7)
		4–21 (0,2,4,6)	5–16 (0,1,3,4,7)	6–Z37e⋆ (0,1,2,3,4,8)
		4–22+ (0,2,4,7)	5–Z18+ (0,1,4,5,7)	6–Z39e⋆ (0,2,3,4,5,8)
			5–24+ (0,1,3,5,7)	6–Z40e (0,1,2,3,5,8)
			5–25 (0,2,3,5,8)	
			5–Z36+ (0,1,2,4,7)	

Key: **Bold**: Subsets of the genus-defining hexachord
⋆: Sets exclusive to the genus
†: Subsets of 8–28 (0,1,3,4,6,7,9,10), the octatonic scale, but not of 6–30 (0,1,3,6,7,9)
{}: Tentative assignments to genera
+: Gregarious set classes (with the highest genus membership)
e: Membership of Eriksson's genera

'a relatively simple procedure that generates a whole system'.[62] Rules have now been set out governing the formation of the genera in order to incorporate unifying aspects such as inclusion relations and the generation of a genus from one or more initial set classes, thus satisfying elements of key criteria 2 and 9: that a potential 'quality space' (unified spatial model) should be determined by a 'hierarchical taxonomy' of species or genera, and that consideration should be given to subset structure.

Just two deviant set classes, the complementary pair 5–Z12 (0,1,3,5,6) and 7–Z12 (0,1,2,3,4,7,9), stubbornly refuse to belong to any genus; they will therefore have to be placed tentatively in braces ({}) in Table 8 within those genera to which their Z-partners, 5–Z36 (0,1,2,4,7) and 7–Z36 (0,1,2,3,5,6,8), and their hexachordal supersets/subsets belong.[63] Two other set classes, 2–2 (0,2) and 2–4 (0,4), have been placed in braces into the Bichromatic genus, indicating their nominal status as subsets (and the nominal status of their complements, set classes 10–2 and 10–4, as supersets) of the progenitor hexachord, 6–7 (0,1,2,6,7,8), despite their inability to form part of any bichromatic BIG chain for this system, as witnessed by their absence from Table A2. For a similar reason, set class 2–4 (0,4) is also placed in braces in the Hexatonic genus through its status as a subset of 6–20 (0,1,4,5,8,9), despite its absence from the BIG chains of Table A1. Set classes 4–Z15 (0,1,4,6) and 4–Z29 (0,1,3,7) are equivocal, too, owing to the uniquely non-committal nature of their all-interval interval-class vector profiles. They squeak into the Octatonic genus by virtue of their status as subsets of both 6–30 (0,1,3,6,7,9) and the octatonic scale 8–28 (0,1,3,4,6,7,9,10), but actually they fail to engage with most of the other genera by only the narrowest of margins; this jack-of-all-trades-master-of-none peculiarity will be explored later in the article as a proto-generic characteristic shared with several other set classes of neutral allegiance.[64]

Tables A1–A6 show various relationships and forms of sub-categorisation, which allow further abstract equivalences and transformational symmetries to be particularised (satisfying key criteria 6 and 9). An indication is given, for instance, of complement-relation types present within or between particular BIG chains: those labelled '(S)' have a symmetrical growth pattern, commensurate with the consequence that constituent larger sets of cardinalities 7–10 all turn out to be true complements of the BIG chains' smaller sets, while those labelled '(R)' have a reciprocal pattern, where supersets of one Z-designated hexachord are the exact complements of the subsets of its Z-partner, and vice versa.[65] These latter are braced together (}) when they extend beyond their hexachords, and are labelled 'R'. Some octatonic BIG chains are labelled '(S8)' or 'R8', demonstrating that in these instances symmetrical or reciprocal complementation occurs within an eight-note (8–28) aggregate. One further form of categorisation, labelled '(M)' or braced (}) as 'M', indicates that set classes in these (usually braced) BIG chains are related through the cycle-of-fourths (or cycle-of-fifths) transform.[66] All diatonic and chromatic BIG chains are inexorably linked one-to-one from one system to the other through this M_5 or M_7 transform and have therefore been contiguously arranged thus in the adjacently placed Tables A5 and A6. For the same reason, all set classes in the Diatonic and Chromatic genera in Table 8 can be matched one-to-one. The reflective aspect of this phenomenon will be used later as a means towards developing the relational ordering of set classes within a diatonic-to-chromatic spectrum. Finally, some BIG chains are replicated in full from one genus to another, suggesting areas of strong intergeneric linkage (satisfying key criterion 5).[67]

Table 8 conforms closely to Eriksson's generic regions based on mm vectors and at the same time promotes many other set classes to a quasi-prototypical status (i.e. those set classes marked with an asterisk [*] but not previously identified as having genus exclusivity).[68] Unlike Eriksson's regions, however, Table 8 shows many individual set classes occupying more than one genus, to some extent contributing to a prismatic filtering of tangible scale- or aggregate-related collections (as in Forte's genera) but also retaining a pragmatic footing in their relational links with incipient prototype hexachords (as in Eriksson's regions and Parks's first four 'Debussy' genera), thereby satisfying key criteria 5 and 8: that the system as a whole would accommodate overlapping regions but that each genus would be organised around its own prototype(s). Indeed, my six genera represent expanded forms of Eriksson's regions, while the Octatonic, Whole-Tone, Diatonic and Chromatic parts of my system represent expanded forms of Parks's like-named 'Debussy' genera, and all but my Whole-Tone bear some relation to Forte's genera.[69] Whereas Forte's trichord-generated genera all have low representations of trichords and tetrachords (with correspondingly low duplications between genera) and unavoidably higher incidences of hexachords (with correspondingly higher duplications between genera), the burden of regulation in Table 8 has shifted to the hexachords, which now become the least duplicated and least represented category within each genus, while incipient sets of low cardinality have become correspondingly higher in their duplication and greater in their incidence within each genus.[70] All sets focused in one genus as distant 'loners', marked with an asterisk in Table 8, and all subsets of a defining prototypical hexachord, shown in boldface type in Table 8, are 'characteristic' of the genus, intuitively occupying a spatial area of affinity between the prototype itself and those set classes shared by more than one genus.[71] Gregarious, 'friendly' or 'sociable' set classes are labelled '+' in Table 8, while set classes which correspond to Eriksson's regions are labelled 'e'. Although most intergeneric links are provided by pentachords and hexachords, in some circumstances dyads, trichords and tetrachords can just as easily provide definitive areas of two-way and three-way linkage. More generally, though, it is the mid-sized tetrachord category of set classes (and the eight-element complements) that proves to be the most genus specific of the cardinalities. A survey of the 29 tetrachordal set classes shows that, when considered as true subsets of the prototypical hexachords, almost every one of them is distributed individually and exclusively into one genus, the scant 'social' exceptions being 4–9 (0,1,6,7), 4–25 (0,2,6,8) and the diatonic/chromatic 4–10 (0,2,3,5) and 4–11 (0,1,3,5).

Following the approach of Forte and Parks, attention can now be focused on the definition of interpretative rules by which one genus is given preference over other genera in the analysis of any particular piece or passage, in order to deal more fully with intergeneric affinities (satisfying key criterion 5).[72] As a first principle, set classes whose entries are in bold or are marked with an asterisk (*) or dagger (†) in Table 8 ought to take precedence over other set-class entries as analytical pointers, since they represent subsets of the defining hexachord or scale, or set classes that are exclusive to the genus. But just as interesting for analytical practice, perhaps, are those more neutral set classes which only marginally associate with two or more genera and which show no particular allegiance to any. An example in this category would be set class 5–11 (0,2,3,4,7), which has an intriguingly even spread of ics 1 to 5 in its interval-class vector [222220] and a confusing range of membership spanning the Hexatonic, Diatonic and Chromatic genera.[73] Clearly, contextual association is essential in such cases: aligned set classes in Forte's analysis of the third of Schoenberg's *Five Orchestral Pieces*, Op. 16, for example, help to place these instances of set class 5–11 firmly in the Hexatonic camp, albeit with some intriguing sideways glances towards the Diatonic and Chromatic.[74] Furthermore, the analytical method has to be flexible enough to reflect perceived overall patterns of pitch organisation as well as more localised shifts and relationships. Isaacson has emphasised 'the crucial role [that] segmentation plays in analysis of this sort' and warns that '[t]he identification of musical segments can have a powerful effect on similarity relations'.[75] He issues a timely reminder that contrast should be seen to feature just as strongly as similarity in music of the atonal era, and that 'the tension between similarity and contrast is central to the way much western art music works'.[76] He shows how different segmentations of a melodic phrase from the fourth of Schoenberg's *Six Little Piano Pieces*, Op. 19, can highlight different relational aspects, revealing a shimmering and multi-faceted assemblage of generic components.

Forte's segmentation of the third of Webern's *Four Pieces* for violin and piano, Op. 7, is bound together by a pervasive 4–9 (0,1,6,7) motive, interacting with the other complexes around it.[77] This motive is initially incorporated within a Bichromatic ambit of set classes but is subsequently aligned within an Octatonic one.[78] Such neat divisions between the genera are, of course, something of a rarity in practice. Indeed, it has often been proposed that differing scale and aggregate collections frequently correlate or coexist in particular ways. Richard Cohn, for instance, advises that 'interpenetrations of diatonic and hexatonic principles suggest that the hexatonic model is likely to achieve the broadest scope and deepest insight into nineteenth-century music if used not in isolation from standard diatonic models, but rather in conjunction with them ... . Although extended triadic progressions may simultaneously be interpretable in diatonic and hexatonic (or more generally, chromatic) spaces, ... "phenomena hearable in two or more ways" function as [mediating] pivots ... between hexatonic and diatonic [or chromatic] space'.[79] Forte likewise warns that 'we do well to regard "the octatonic" as a network with multiple connections to the other harmonic environments, and not merely as an ordered scalar pattern'.[80] Ultimately, segmentation will reveal subtle changes between genera, sometimes involving shared sets, and sometimes across much smaller time spans of perhaps only one or two bars. As an example, the piano chords in bars 4–6 of the second of Berg's *Four Pieces* for clarinet and piano, Op. 5, intermingle Octatonic (or almost-Octatonic), Diatonic and Hexatonic chords with Octatonic and Chromatic lateral lines. This occurs through the medium of a profuse array of Octatonic/Diatonic/Hexatonic 4–27s (0,2,5,8) before conceding to the Hexatonic collection, 6–20 (0,1,4,5, 8,9), at the end of the sequence.

IV. A Detour Demonstrating the Ameliorative Effect of the M_5 Transform in Creating a Diatonic-to-Chromatic Spectrum of All Set Classes, Followed by a Foray into Interval-Class Vector Characteristics as a Determinant for Genus-Based Levels of Set-Class Distance, Finally to Gain Access to an Affinity Mapping of the Set-Class Universe

The next step forward in this quest for a three-dimensional representation of all set classes will involve the creation of an all-inclusive (one-dimensional) diatonic-to-chromatic ordering, which can then be expanded into three dimensions.[81] This will be achieved by employing multiplicative relationships between set classes, specifically those derived from operations which augment elements of the prime forms of set classes by a factor of five or seven. Although these operations (often called 'M_5' and 'M_7') have traditionally been used as compositional strategies for transforming row segments or tropes, the resultant set-class relationships can provide useful analytical insights, and can have far-reaching theoretical consequences as an organisational means of placing set classes in a multi-dimensional space. Under these operations, elements of the prime form of a set class (pitch or interval) are multiplied by either five or seven (i.e. exponentially transposed or widened by these multipliers). The operation creates an equivalent set class which is either another (T_n or T_nI) instance of the same prime form of a set class (i.e. an M-invariant partner, formed through a self-mapping non-transformation) or a related set class of the same cardinality and with the same degree of symmetry (that is, an M-related partner, formed through a reciprocally mapping transformation).[82] Put another way, the fourths/fifths-cycle aspect of a set class is 'swapped' for the chromatic-cycle aspect under the operation, or vice versa. Fig. 4a shows how interval-class vector multiplicities of ics 2, 3, 4 and 6 (and their inversions, 10, 9, 8 and 6) are inverted or retained intact when multiplied by five or seven, while those of ics 1 and 5 (and their inversions, 11 and 7) are swapped or mapped into each other. Fig. 4b gives an example of the application of the operation(s) to a set class in order to transform it into a reciprocal (M-related) set class. Fig. 4c gives an example of the alternative application of the operation(s) to a set class in order to create an instance

Fig. 4a The conversion of the interval 1 cycle to the interval 5 or interval 7 cycle by multiplying each element by 5 or 7, mod. 12 (transposing each element by T_5 or T_7)

ic 1 cycle	0	1	2	3	4	5	6	7	8	9	10	11
×5↓												
ic 5 cycle	0	5	10	3	8	1	6	11	4	9	2	7

ic 1 cycle	0	1	2	3	4	5	6	7	8	9	10	11
×7↓												
ic 7 cycle	0	7	2	9	4	11	6	1	8	3	10	5

Fig. 4b An example of the M_5/M_7 transform of a set class to its M-related set class

set class 4–27 (0,2,5,8), ic vector [**0**121**1**1] with ics 1 and 5 in bold

prime form (0,2,5,8) × 5 or × 7 = (0,10,25,40) or (0,14,35,56)

= mod 12 (0,10,1,4) or (0,2,11,8)

= prime form (0,2,3,6) set class 4–12, ic vector [**1**121**0**1]

Fig. 4c An example of the M_5/M_7 transform of a set class to its M-invariant self

set class 4–10 (0,2,3,5), ic vector [**1**220**1**0]

prime form (0,2,3,5) × 5 or × 7 = (0,10,15,25) or (0,14,21,35)

= mod 12 (0,10,3,1) or (0,2,9,11)

= prime form (0,2,3,5) set class 4-10, ic vector [**1**220**1**0]

Fig. 4d An example of the reapplication of the M_5/M_7 transform of the M-related equivalent set class, in order that the original set class is restored

set class 4–12 (0,2,3,6), ic vector [**1**121**0**1]

prime form (0,2,3,6) × 5 or × 7 = (0,10,15,30) or (0,14,21,42)

= mod 12 (0,10,3,6) or (0,2,9,6)

= prime form (0,2,5,8) set class 4–27, ic vector [**0**121**1**1]

of the same (M-invariant) set class; in this, and in any other operation involving M invariants, a 'hidden' swap occurs, simply because such set classes already happen to have the same number of ics 1 and 5. Either operation, M_5 or M_7, will produce the same transformational or equivalent set class. When either operation is re-applied to the M-related equivalent set class, the original set class is restored. Fig. 4d gives an example of this reverse operation.

With M-invariant set classes, the equivalences are often both T_n and T_nI related under the operation, on account of their inherent symmetry. M-related set classes clearly differ in this respect: only when an original set class is reproduced through a second multiplicative step can it be seen that this new version

Fig. 5 Berg, Op. 5: cyclic set class patterns
(a) T_8 and T_4 cyclic patterns of instances of set class 6–Z44 (0,1,2,5,6,9)
(b) T_4 cyclic pattern of instances of set class 6–Z19 (0,1,3,4,7,8)

(a)

I/1, T_0 $\xrightarrow{T_8}$ I/9, T_8 $\xrightarrow{T_8}$ III/1, T_4 $\xrightarrow{T_8}$ III/14–18, T_0
IV/2 and IV/6–18, T_0

I/9, T_8 $\xrightarrow{T_4}$ II/1–2, T_0 $\xrightarrow{T_4}$ III/1, T_0

(b)

II/1–4, T_0 $\xrightarrow{T_4}$ I/9, T_4 $\xrightarrow{T_4}$ I/6–7, T_8 $\xrightarrow{T_4}$ IV/3, T_0

of the original set class has been transposed, at one of four possible levels: T_0, T_6, T_4 or T_8. In the case of the latter three transpositions, a $T_4 + T_4 + T_4 = T_0$ or $T_8 + T_8 + T_8 = T_0$ process (for M_5 operations) or a $T_6 + T_6 = T_0$ process (for M_7 operations) will be required of either of the set classes involved before the original transpositional level is restored. Transpositional cycles of this type can be demonstrated in Berg's Op. 5, where there are numerous prominent and interrelated instances of the Z/M-related pair 6–Z19 (0,1,3,4,7,8) and 6–Z44 (0,1,2,5,6,9), most of them occurring at the start and end of each of the four pieces. These instances will fit into T_4 and T_8 cyclic processes: those for 6–Z44 follow temporal paths, as shown in Fig. 5a, while the one for 6–Z19 is less clearly defined compositionally, as shown in Fig. 5b. Consequently, a sixfold M_5 cycle of transformationally interrelated 6–Z44s and 6–Z19s can be set up incorporating the T_4 cycles for each hexachord. The pitch-class correlations highlighted by this (theoretical) sixfold cycle are not entirely accidental, given Berg's predilection for many of these pitch classes at the beginnings and ends of the four pieces. Although theoretically there are seven other sixfold M_5 cycles involving interrelated 6–Z44s and 6-Z19s and incorporating other pitch-class correlations, only this particular one predominates in Op. 5.[83]

The M_5/M_7 procedure is of particular interest to the analyst because it throws up a very different division of the set classes. These are shown in Table 9 for cardinals 2 to 6 only. The centrally placed groups of M-invariant (self-mapped) set classes for each cardinality (53 in all) includes eight Z-hexachords and two Z-pentachords which, in this context, now subsist as autonomous entities, unconnected to their Z-partners, while the symmetrically, more remotely placed group of M-related (reciprocally mapped) pairs for each cardinality, labelled 'a', 'b', and so on, now incorporates a mixture of non-Z, Z-related and mixed-Z pairings. All set-class entries in Table 9 show their ics 1/5 content and their ics 2/3/4/6 content separated into two columns. In retrospect, we can now see that

Table 9 The M_5/M_7-related pairs of set classes (labeled 'a', 'b', etc.) and M_5/M_7-invariant set classes, all with their ics 1/5 and ics 2/3/4/6 interval-class vector content separated

Cardinals 2, 3 and 4	ics 1/5	ics 2/3/4/6
+ 2–1 a	[10]	[0000]
2–2	[00]	[1000]
2–3	[00]	[0100]
2–4	[00]	[0010]
2–6	[00]	[0001]
+ 2–5a	[01]	[0000]
++ 3–1a	[20]	[1000]
+ 3–2 b	[10]	[1100]
+ 3–3 c	[10]	[0110]
3–4	[11]	[0010]
3–5	[11]	[0001]
3–6	[00]	[2010]
3–8	[00]	[1011]
3–10	[00]	[0201]
3–12	[00]	[0030]
+ 3–11 c	[01]	[0110]
+ 3–7 b	[01]	[1100]
++ 3–9a	[02]	[1000]
+++ 4–1a	[30]	[2100]
++ 4–2 b	[20]	[2110]
++ 4–3 c	[20]	[1210]
+ 4–4 d	[21]	[1110]
+ 4–5 e	[21]	[1011]
+ 4–7 f	[21]	[0120]
+ 4–12 g	[10]	[1211]
4Z15 h	[11]	[1111]
4–6	[22]	[1001]
4–8	[22]	[0011]
4–9	[22]	[0002]
4–10	[11]	[2200]
4–11	[11]	[2110]
4–13	[11]	[1201]
4–17	[11]	[0220]
4–18	[11]	[0211]
4–19	[11]	[0130]
4–21	[00]	[3021]
4–24	[00]	[2031]
4–25	[00]	[2022]
4–28	[00]	[0402]
4–Z29 h	[11]	[1111]
+ 4–27 g	[01]	[1211]
+ 4–20 f	[12]	[0120]
+ 4–16 e	[12]	[1011]
+ 4–14 d	[12]	[1110]
++ 4–26 c	[02]	[1210]
++ 4–22 b	[02]	[2110]
+++ 4–23 a	[03]	[2100]

Cardinal 5	ics 1/5	ics 2/3/4/6
++++ 5—1a	[40]	[3210]
++ 5–2 b	[31]	[3210]
++ 5–3 c	[31]	[2220]
++ 5–4 d	[31]	[2211]
++ 5–8 e	[20]	[3221]
+ 5–5 f	[32]	[2111]
+ 5–6 g	[32]	[1121]
+ 5–9 h	[21]	[3121]
+ 5–10 i	[21]	[2311]
+ 5–13 j	[21]	[2131]
+ 5–16 k	[21]	[1321]
5–Z17 l	[22]	[1230]
5–Z18 m	[22]	[1221]
5–7	[33]	[1012]
5–11	[22]	[2220]
5–Z12	[22]	[2211]
5–Z36	[22]	[2211]
5–15	[22]	[2022]
5–19	[22]	[1212]
5–21	[22]	[0240]
5–22	[22]	[0231]
5–26	[11]	[2231]
5–28	[11]	[2222]
5–31	[11]	[1412]
5–33	[00]	[4042]
5–Z38 m	[22]	[1221]
5–Z37 l	[22]	[1230]
+ 5–32 k	[12]	[1321]
+ 5–30 j	[12]	[2131]
+ 5–25 i	[12]	[2311]
+ 5–24 h	[12]	[3121]
+ 5–20 g	[23]	[1121]
+ 5–14 f	[23]	[2111]
++ 5–34 e	[02]	[3221]
++ 5–29 d	[13]	[2211]
++ 5–27 c	[13]	[2220]
++ 5–23 b	[13]	[3210]
++++ 5–35 a	[04]	[3210]

Cardinal 6	ics 1/5	ics 2/3/4/6
++++ 6—1 a	[51]	[4320]
+++ 6–2 b	[41]	[4321]
++ 6–Z3 c	[42]	[3321]
++ 6–Z4 d	[42]	[3231]
++ 6–Z36 e	[42]	[3321]
++ 6–Z37 f	[42]	[3231]
+ 6–5 g	[43]	[2222]
+ 6–Z10 h	[32]	[3331]
+ 6–Z39 i	[32]	[3331]
+ 6–15 j	[32]	[2341]
+ 6–Z13 k	[32]	[2422]
+ 6–Z42 l	[32]	[2422]
+ 6–21 m	[12]	[4242]
6–Z6 n	[44]	[2122]
6–Z19 o	[33]	[1341]
6–Z11 p	[33]	[3321]
6–7	[44]	[2023]
6–8	[33]	[4320]
6–9	[33]	[4221]
6–Z12	[33]	[3222]
6–14	[33]	[2340]
6–16	[33]	[2241]
6–Z17	[33]	[2232]
6–20	[33]	[0360]
6–Z41	[33]	[3222]
6–Z43	[33]	[2232]
6–22	[22]	[4142]
6–Z23	[22]	[3422]
6–27	[22]	[2522]
6–Z28	[22]	[2432]
6–30	[22]	[2423]
6–Z45	[22]	[3422]
6–Z49	[22]	[2432]
6–35	[00]	[6063]
6–Z40 p	[33]	[3321]
6–Z44 o	[33]	[1341]
6–Z38 n	[44]	[2122]
+ 6–34 m	[21]	[4242]
+ 6–Z29 l	[23]	[2422]
+ 6–Z50 k	[23]	[2422]
+ 6–31 j	[23]	[2341]
+ 6–Z24 i	[23]	[3331]
+ 6–Z46 h	[23]	[3331]
+ 6–18 g	[34]	[2222]
++ 6–Z48 f	[24]	[3231]
++ 6–Z47 e	[24]	[3321]
++ 6–Z26 d	[24]	[3231]
++ 6–Z25 c	[24]	[3321]
+++ 6–33 b	[14]	[4321]
++++ 6–32 a	[15]	[4320]

Note: The number of plus (+) symbols equals the difference between ics 1 and 5.

M_5/M_7 relations have been pervasive through much of the discourse of this article so far. In Table 5, for instance, generated M-related set classes are cross-referenced between 20 of the 30 combination cycles.[84] Similarly, in Table 6 (the hexachord families), Tables A1–A6 (the BIG chains arranged by genus) and Table 8 (the lists of genera), M-related set classes always appear together within Hexatonic, Bichromatic, Whole-Tone and Octatonic lists, and are either opposed one-to-one or correlated in the Diatonic and Chromatic.[85]

Despite Forte's insistence that his twelve genera display a metaphorical spectrum, from the 'traditional' or 'ancient' diatonic (genera 11 and 12) to the 'exotic' or 'modern' atonal (genera 1, 2 and 3), the M relation can nevertheless be shown to permeate even his 'context insensitive' system.[86] By signalling that there are 'marked similarities between diatonic and chromatic collections', as distributed between his genera, he has more than hinted at such an arrangement, although he has not explicitly described it.[87] His diatonic-to-exotic spectrum can be reinterpreted alternatively as a diatonic-to-chromatic spectrum, where the more 'atonal' areas inform, or are informed by, the diatonic and chromatic ones.[88] It would seem, then, that the M_5 procedure will inevitably give rise to an invaluable hierarchical 'spread' of all set classes, furnishing a distinct shape or profile to the whole set-class universe as well as to each of the genera.[89] Quinn has pointed out that the M operation 'brings together species [set classes] that play the *same* role in two *different* qualitative genera [diatonic and chromatic] that are wholly M-related to one another', in other words, as exemplified by the Diatonic and Chromatic genera set out in Table 8.[90] Moreover, it is clear that all M-invariant, self-mapping set classes are either both Diatonic *and* Chromatic or neither.[91] So, since M_5 equivalences have been produced through the cycle of fourths/fifths transform, we are in a way looking at all set classes though a diatonic-to-chromatic filter or prism of affinity. All set classes take their positions as part of an all-inclusive spectrum ranging from purely diatonic to purely chromatic, a spectrum which embraces not only the diatonic and chromatic 'crossover' set classes, but also the members of the other genera on the way, extending between the diatonic and chromatic extremes.[92]

In order to justify the diatonic-to-chromatic spectrum, as shown for each cardinality in Table 9, we can turn to Samplaski's set-class mappings, as demonstrated in two of his dimensional histograms: his figs 7 and 11, which both show diatonic-to-chromatic spectra for all set classes of cardinals 3, 4 and 5, ordered according to 'ic1-saturation vs. ic5-saturation'.[93] Not only have the constituent set classes from all M-related pairs been separated, they have been symmetrically and systematically spread throughout. The similarity/dissimilarity relations within these symmetrically distributed scalings can be translated into tangible degrees of affinity and distance between set classes. The M-related set classes with the widest difference between ics 1 and 5 are placed at the extremes of the scalings, followed by those slightly less so, and so on, with the M-invariant set classes at the centre. Samplaski has observed this phenomenon precisely, although without specifically referring to the M_5 relation.[94] He finds that, in both methods of scaling, a large number of the sets are clustered closely together around the central zero-point (i.e. the M-invariant set classes that are neither particularly similar nor dissimilar).[95]

Samplaski's other histograms, showing scalings from one typicality to another, prove less useful for uncovering a spread of affinity within any of the other four genera.[96] For a consideration of the totality of all six genera, then, it will be necessary to change direction somewhat and look afresh at the particular interval-class characteristics of each genus, as epitomised by the prototypical hexachords shown in Table 2. Thus, as already demonstrated, Diatonic set classes could be characterised by high ic 5 versus (minus) low ic 1, and Chromatic by high ic 1 versus (minus) low ic 5.[97] Beyond this, Hexatonic set classes are characterised by high ic 4 versus (minus) low ics 2 and 6; Bichromatic by high ics 1, 5 and 6 versus (minus) low ics 2, 3 and 5; Octatonic by high ics 3 and 6 versus (balanced against) an even distribution of ics 1, 2, 4 and 5; and Whole-Tone by high ics 2, 4 and 6 versus (minus) low ics 1, 3 and 5. Scalings for all set classes of cardinals 2 to 6 can be worked out for each genus based on these characteristics.[98] Because these scalings turn out to range between eight and fifteen affinity levels, a result of the application of the different interval-class parameters to each genus, for the sake of consistency I have decided to place set classes on a reduced platform of seven levels for each genus so that relative distances can be judged and cross-generational comparisons can most effectively be made. This has meant that in each case some adjacent levels, particularly those at the most typical end of the spectrum, have had to be conflated.[99] The level of affinity of any set class to each genus prototype can now be set out, as illustrated in the six columns of genera in Table 10, such that a value of 1 denotes maximum affinity to the prototype, a value of 7 denotes minimum typicality, and so on. In particular, level 4 represents those central set classes that are most indifferent or neutral. As might be expected, it can again be observed that M-related set classes are valued identically, except that diatonic and chromatic affinities are reversed, level 1 translating to 7, level 2 to 6 and level 3 to 5.[100] Table 10 allows the measurement of distance between all set classes to be gauged either vertically (i.e. intragenerically for all set classes across each individual genus, both against each other and against the prototypical hexachord) or horizontally (i.e. intergenerically for each set class across all six genera) by noting the difference between numerical values.[101] It can be seen that, in nearly all cases, genus members (those with numerical values in bold) rate appropriately low, confirming their affinity leanings. We can also note that the most 'neutral' set classes with the largest number of values at or close to 4 correlate closely with the ones that have an almost even spread of ics within their ic vectors.[102]

Taken together, the M-related lists (Table 9) and the affinity scalings (Table 10) provide ample further evidence to support several of the key criteria required for the spatial organisation and effective relational arrangement of a system of genera: closely and distantly related sonorities (key criterion 1), 'quality space' determined by similarity relations through the (computational)

Table 10 Genus scalings from most typical (1) to least typical (7) based on the characteristic interval-class content of each prototypical hexachord

	C	W	O	H	D	B
6–1	**1**	7	5	5	7	5
6–2	**1**	5	4	7	7	5
6–Z3/6–Z36	**2**	7	3	5	6	5
6–Z4/6–Z37	**2**	5	4	4	6	5
6–5	3	7	**3**	5	5	**2**
6–Z6/6–Z38	4	7	4	5	4	**2**
6–7	4	5	4	7	4	**1**
6–8	**4**	7	4	5	**4**	5
6–9	**4**	5	4	7	**4**	5
6–Z10/6–Z39	**3**	5	3	4	5	5
6–Z11/6–Z40	**4**	7	3	5	**4**	5
6–Z12/6–Z41	4	5	**3**	7	4	**3**
6–Z13/6–Z42	3	7	**2**	5	5	5
6–14	**4**	7	4	**2**	**4**	5
6–15	**3**	5	3	**2**	5	5
6–16	4	5	4	**2**	4	5
6–Z17/6–Z43	4	5	**3**	4	4	**3**
6–18	5	7	**3**	5	3	**2**
6–Z19/6–Z44	4	7	4	**2**	4	5
6–20	4	7	6	**1**	4	5
6–21	3	**2**	**4**	5	5	6
6–22	4	**2**	4	5	4	**5**
6–Z23/6–Z45	4	5	**2**	7	4	5
6–Z24/6–Z46	5	5	3	4	**3**	5
6–Z25/6–Z47	6	7	3	5	**2**	5
6–Z26/6–Z48	6	5	4	4	**2**	5
6–27	4	7	**1**	5	4	5
6–Z28/6–Z49	4	5	**2**	4	4	5
6–Z29/6–Z50	5	7	**2**	5	3	5
6–30	4	5	**1**	7	4	5
6–31	5	5	3	**2**	**3**	5
6–32	7	7	5	5	**1**	5
6–33	7	5	4	7	**1**	5
6–34	5	**2**	**4**	5	3	6
6–35	4	**1**	7	7	4	7

	C	W	O	H	D	B
5–1	**1**	6	5	5	7	4
5–2	**2**	6	4	5	6	4
5–3	**2**	6	4	**3**	6	4
5–4	**2**	6	**3**	5	6	**3**
5–5	**3**	6	4	5	5	**3**
5–6	**3**	6	**4**	**3**	5	**3**
5–7	4	6	**4**	5	4	**1**
5–8	**2**	**3**	4	5	6	5
5–9	**3**	**3**	4	5	**5**	**4**
5–10	**3**	6	**2**	5	**5**	4
5–11	**4**	6	4	**3**	**4**	5
5–Z12/5–Z36	**4**	6	**3**	5	**4**	**4**
5–13	3	**3**	4	**3**	5	**4**
5–14	5	6	4	5	**3**	**3**
5–15	4	3	**4**	5	4	**3**
5–16	**3**	6	**2**	**3**	5	4
5–Z17/5–Z37	4	6	4	**2**	4	4
5–Z18/5–Z38	**4**	6	**3**	**3**	4	**4**
5–19	4	6	**2**	5	4	**3**
5–20	5	6	**4**	**3**	**3**	**3**
5–21	4	6	5	**1**	4	4
5–22	4	6	4	**2**	4	4
5–23	6	6	4	5	**2**	4
5–24	**5**	**3**	4	5	**3**	**4**
5–25	**5**	6	**2**	5	**3**	4
5–26	4	**3**	**3**	**3**	4	5
5–27	6	6	4	**3**	**2**	4
5–28	4	**3**	**2**	5	4	**4**
5–29	6	6	**3**	5	**2**	**3**
5–30	5	**3**	4	**3**	3	**4**
5–31	4	6	**1**	5	4	4
5–32	5	6	**2**	**3**	**3**	4
5–33	4	**1**	6	5	4	6
5–34	6	**3**	4	5	**2**	5
5–35	7	6	5	5	**1**	4

	C	W	O	H	D	B
4–1	**1**	6	4	5	7	**4**
4–2	**2**	**4**	4	**4**	**6**	**4**
4–3	**2**	6	**4**	3	6	4
4–4	**3**	6	**4**	**3**	5	**4**
4–5	**3**	4	**4**	**3**	5	**3**
4–6	4	6	4	5	4	**2**
4–7	**3**	6	4	**2**	5	4
4–8	4	6	4	3	4	**2**
4–9	4	6	**4**	5	4	**1**
4–10	**4**	6	**4**	5	**4**	5
4–11	**4**	**4**	4	4	**4**	5
4–12	**3**	**4**	**3**	**3**	5	4
4–13	**4**	6	**3**	5	**4**	**4**
4–14	5	6	**4**	**3**	**3**	**4**
4–Z15/4–Z29	4	4	**4**	4	4	4
4–16	5	4	**4**	**3**	**3**	**3**
4–17	4	6	**4**	**2**	4	4
4–18	**4**	6	**3**	**3**	**4**	**4**
4–19	4	**4**	5	**1**	4	5
4–20	5	6	4	**2**	**3**	4
4–21	**4**	**2**	4	5	**4**	5
4–22	**6**	**4**	4	**4**	**2**	**4**
4–23	7	6	4	5	**1**	**4**
4–24	4	**2**	4	**3**	4	5
4–25	4	**2**	**4**	5	4	**4**
4–26	6	6	**4**	3	**2**	4
4–27	5	**4**	**3**	**3**	**3**	4
4–28	4	6	**1**	5	4	4
3–1	**2**	5	**4**	**4**	6	**3**
3–2	**3**	5	**3**	4	**5**	5
3–3	**3**	5	**3**	**2**	5	5
3–4	**4**	5	**4**	**2**	**4**	**3**
3–5	4	5	**3**	4	4	**3**
3–6	**4**	**2**	4	4	**4**	5
3–7	**5**	5	**3**	4	**3**	5
3–8	4	**2**	**3**	4	4	**5**
3–9	6	5	**4**	**4**	**2**	**3**
3–10	**4**	5	**1**	**4**	**4**	5
3–11	5	5	**3**	**4**	**3**	5
3–12	4	**2**	4	**1**	4	5
2–1	**3**	5	**2**	**3**	**5**	**4**
2–2	**4**	**3**	**2**	4	**4**	**5**
2–3	**4**	5	**2**	**3**	**4**	5
2–4	**4**	**3**	**2**	**2**	**4**	**5**
2–5	**5**	5	**2**	**3**	**3**	**4**
2–6	4	**3**	**2**	4	4	**4**

Key: C = Chromatic
W = Whole-tone
O = Octatonic
H = Hexatonic
D = Diatonic
B = Bichromatic

Note: Numerals in bold represent genus members.

application of a numerical index and a hierarchical taxonomy (key criterion 2), the relation of sonorities to distances in space (key criterion 4), the intrageneric and intergeneric affinities of sets within and between regions (key criterion 5), the inherence of set-class quality with set-class equivalence in both their interval content and transformational symmetries (key criterion 7) and the spatial structuring of set-class affinities through qualitative closeness to or distance from the genus prototypes (key criterion 9). The precise spatial organisation and relational arrangement of all set classes are, however, dependent on the final stage of this odyssey: the invention of an appropriate spatial model.

V. Looking at Some Multi-Dimensional Models, Finally Arriving at the Creation and Plotting of a Systematised Three-Dimensional Set-Class Space

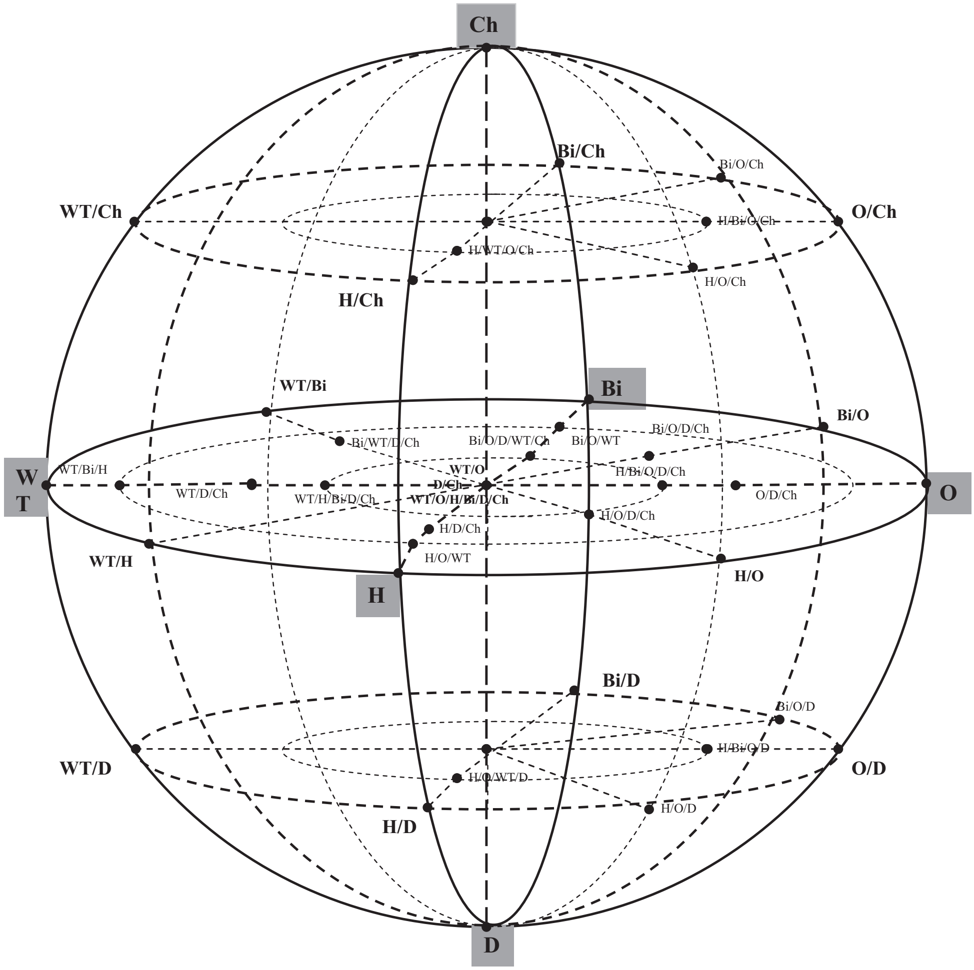
We can start this final stage of the odyssey by attempting to satisfy key criterion 8: that genus prototypes should occupy distant points in set-class space. Just as the M_5-induced spectra of Table 9 (and Samplaski's diatonic-to-chromatic histograms) are one-dimensional, so they can be arranged as a one-dimensional axis enclosed at its poles by the diatonic and chromatic prototype hexachords, 6–32 (0,2,4,5,7,9) and 6–1 (0,1,2,3,4,5). Atte Tenkanen has foreseen a problem in graphically representing set classes such as 6–1 (0,1,2,3,4,5), 6–32 (0,2,4,5,7,9) and 6–8 (0,2,3,4,5,7), since they may end up occupying the same space owing to identical ics 2, 3, 4 and 6 in their interval-class vectors (i.e. [5**4321**0], [1**4325**0] and [3**4323**0] respectively).[103] My intention to give polar placements to set classes 6–32 (0,2,4,5,7,9) and 6–1 (0,1,2,3,4,5) indicates that they are at once both opposites and mirror images; it follows that a central placement on the Diatonic-to-Chromatic axis for set class 6–8 (0,2,3,4,5,7) would indicate that it shares Diatonic and Chromatic interval-class attributes in equal measure. Other set classes placed on this spectrum would either be M-related, mirrored set classes or would, like 6–8, occupy the central, balancing M-invariant point, either as Chromatic and Diatonic representatives or as neither.

To date, a few two- and three-dimensional models have been proposed for a wider group of genera. Eriksson's graphic representation of his seven regions was the first of these, displaying three main areas.[104] More recently, Quinn has offered three illustrations of intergeneric and intrageneric affinities. The first is a triangular arrangement taken from Morris.[105] The second is a Cartesian plane modelling a chromatic-to-diatonic transition between seven 'harmonic states' taken from a piece by Ligeti.[106] Quinn's third illustration is of a hexatonic-to-whole-tone progression of eleven hexachords, all of which have a high ic 4 content and incorporate R_p, R_1, R_2 and M_5 similarity relations.[107] Two-dimensional voice-leading spaces have been provided by Callender, Quinn and Tymoczko for trichords and tetrachords, while three-dimensional models have been devised by Straus, illustrating voice-leading spaces for each of the cardinalities and for adjacent cardinalities.[108] Cohn has designed an interactive tetrahedral example, based on one of Straus's models, showing voice-leading similarities between the 29 tetrachord set classes; this is also represented three-dimensionally in Callender, Quinn, and Tymoczko.[109] Finally, Tenkanen has produced a three-dimensional graph comparing a selection of eighteen trichords and tetrachords.[110]

A multi-dimensional model capable of showing intergeneric correlations has not been attempted before now. Cohn highlights the problem when he warns that this would be 'easy to state but hard to intuit, harder yet to explore, yet again to demonstrate'.[111] Nevertheless, Quinn has proposed a way forward through his assertion that '[t]he M operation corresponds to a reflection (flip) about a vertical line running through the center'.[112] This implies that the other four genus prototypes would, as M invariants, have to be positioned centrally on this Diatonic-to-Chromatic 'axis of reflection corresponding to M', since 'the centers of these [four] genera lie directly on the axis of symmetry in question'. It further implies that these four genera would themselves congregate around them in M_5 space, since 'M_5 preserves the *intra*generic affinities of these other four genera'.[113] Cohn has confirmed that these four collections perform an undermining role, since they insinuate their different qualities [spaces] onto the normal diatonic/chromatic ones, thus acting as 'pivots between diatonic and chromatic space'.[114]

My suggestion, then, is that the precise symmetry of the one-dimensional M_5 construct could be used as a basis for the creation of a three-dimensional representation of the complete system of genera. I propose that a productive way towards delivering this would be to grant the four non-diatonic/chromatic foci, 6–7 (0,1,2,6,7,8), 6–20 (0,1,4,5,8,9), 6–30 (0,1,3,6,7,9) and 6–35 (0,2,4,6,8,10), their own spatial 'homes' at distant points on M-invariant axes placed perpendicular to that of the diatonic-to-chromatic axis, making manifest their unique interval-class characteristics and 'lonely' situation within the set-class universe. Their associated families would then encircle and envelop the diatonic-to-chromatic axis three-dimensionally, somewhat like a corona.[115] I suggest that the familiar shape of a transparent globe, shown in Fig. 6, could be the most appropriate means of representing this arrangement in a relatively unambiguous way. The Chromatic and Diatonic prototypes would take the globe's 'north' and 'south' pole positions, while the other four prototypes would take the 'east', 'west', 'close' and distant' positions around the globe's 'equator'. The equatorial horizontal circular plane cutting through the globe represents Quinn's (and Samplaski's) M_5 (M-invariant) axis of symmetry.[116] The Hexatonic and Whole-Tone nodes are best placed adjacent to each other (i.e. at 90 degrees rotationally), reflecting their mutual derivation from set class 3–12 (0,4,8) origins (see again Table 5), while the Bichromatic and Octatonic nodes can occupy the opposite adjacencies, making manifest the several BIG chains that they have in common.[117] The six genus nodes of Fig. 6 can be considered to be roughly equidistant, thereby reflecting the basic dissimilarity between the six hexachordal progenitors but also allowing the elaboration required for the ultimate placement of all of the set classes to be accommodated.[118]

Fig. 6 Placement of intergeneric points on the globe



Key: Bi: Bichromatic, Ch: Chromatic, D: Diatonic, H: Hexatonic, O: Octatonic, WT: Whole-Tone

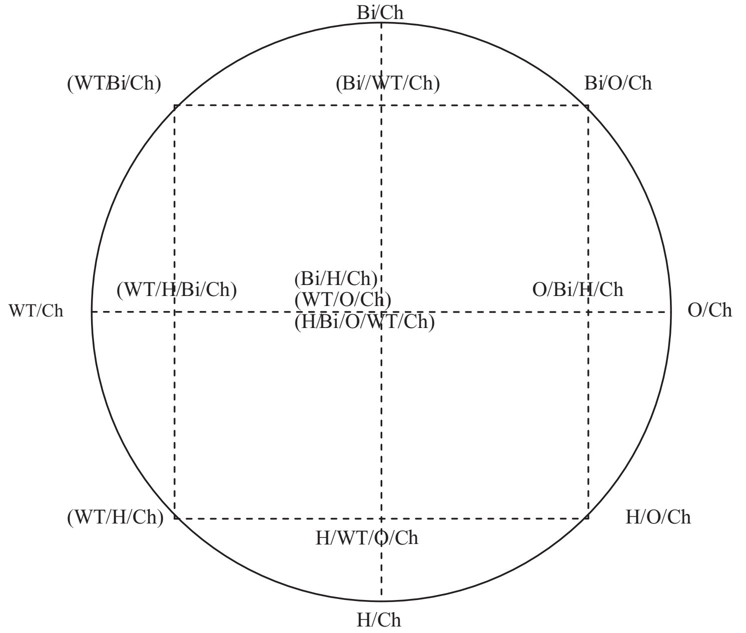
Fig. 6 shows how the six genus nodes form the limit points for three axes passing through the globe, Diatonic to Chromatic, Hexatonic to Bichromatic and Whole-Tone to Octatonic.[119] It also shows three intersecting 'great' circles, two running north to south and back, and one running laterally as the globe's 'equator'. Each of the three great circles crosses four genus nodes, and taken together they connect the six genus nodes in twelve more ways.[120] The total of fifteen two-genus (intergeneric) coordinates implied by these three axial and twelve great-circle connections can most practically be placed midway between each of the fifteen pairings of genus nodes; all but one of these are also shown in Fig. 6.[121] These fifteen two-genus nodes in turn offer the opportunity for eight three-genus points of interface to be placed. Finally, three horizontal circular planes can cut through the globe, a larger one aligned to the equator and the smaller ones slicing through the northern and southern 'hemispheres'. These horizontal planes allow for the placement of a final 34 internal points of interface, of between three and six levels of intersection. While Fig. 6 only shows those placements for points of generic intersection that actually occur between the six genera (as in Table 11), Figs 7a, b and c show all of the potential intergeneric placements on the three horizontal planes (those in parentheses are theoretically but not actually present).[122]

Although it is not possible to place individual set classes according to precisely measured unit distances from a genus 'limit case', as advocated by Quinn through his fuzzified 'lewin' distances (derived from Fourier balances),[123] it will nevertheless now be possible to give fairly accurate placements on and within the globe by collating points of intersection with the criteria presented in Tables 8 and 9. All set classes can then be assigned and located according to their membership, either to one of the 6 genus nodal 'hot spots' or to one of the 57 points of intersection between these genus nodes as they appear in Figs 6 and 7. Although these intersection placements of set classes have the initial disadvantage of being approximate, they nevertheless have the crucial advantage of being automatically assigned within the three-dimensional space. Whereas Quinn's Fourier balance–induced model will produce coordinated placements between just two genera, I have created intersections that allow the coordination of set classes in any number of different intergeneric ways while at the same time creating intrageneric placements as well.[124] Accordingly, nodal points of intersection in many cases represent intergeneric and intrageneric affinities at one and the same time.

Several potential anomalies arise at this stage in the construction, caused by the contraction of the theoretically ideal (but impractical) five- or six-dimensional set-class space to only three dimensions. In practice, these do not present too much of a problem. For instance, although three unrelated two-genus interfaces, three unrelated interfaces involving four genera and the six-genus interface all theoretically occupy the central node of the equatorial horizontal plane (and of the globe), there happens to be no Hexatonic/Bichromatic interface, nor do any of the three theoretical four-genus interfaces actually appear in practice.[125] Again, although theoretically two three-genus interfaces and one five-genus interface all occupy the central node of the northern horizontal plane, fortuitously these interfaces have not materialised, and the node is therefore empty. Similarly, although two three-genus interfaces and one five-genus interface all theoretically occupy the central node of the southern horizontal plane, these interfaces do not exist either, so this node is also conveniently empty. Some of the internal interfaces on the Whole-Tone-to-Octatonic and Hexatonic-to-Bichromatic axes on the equatorial horizontal plane are inevitably crowded together, and in some cases a pragmatic positional choice has had to be made. Nevertheless, these choices do have directional consistency

Fig. 7 'Geographical' planes
(a) 'Northern' plane: 15 points of interface
(b) 'Southern' plane: 15 points of interface
(c) 'Equatorial' plane: 4 genus nodes and 31 points of interface

(a)



(b)

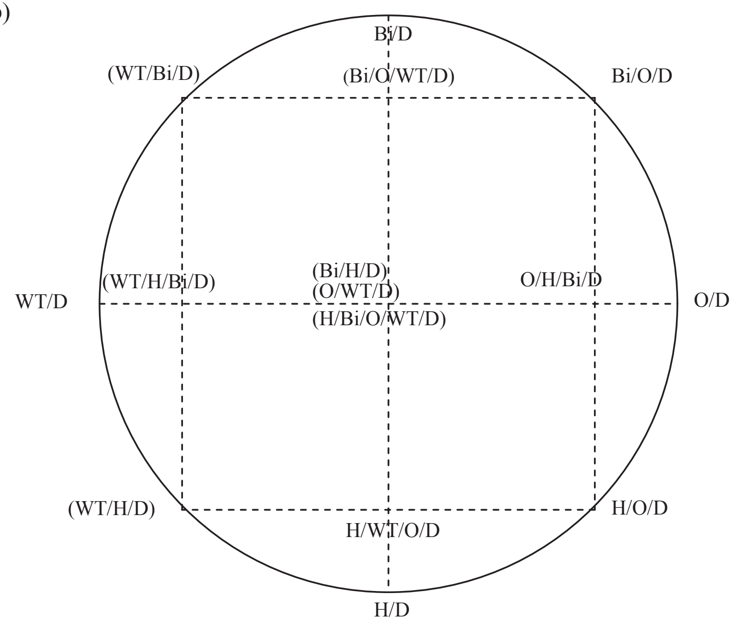
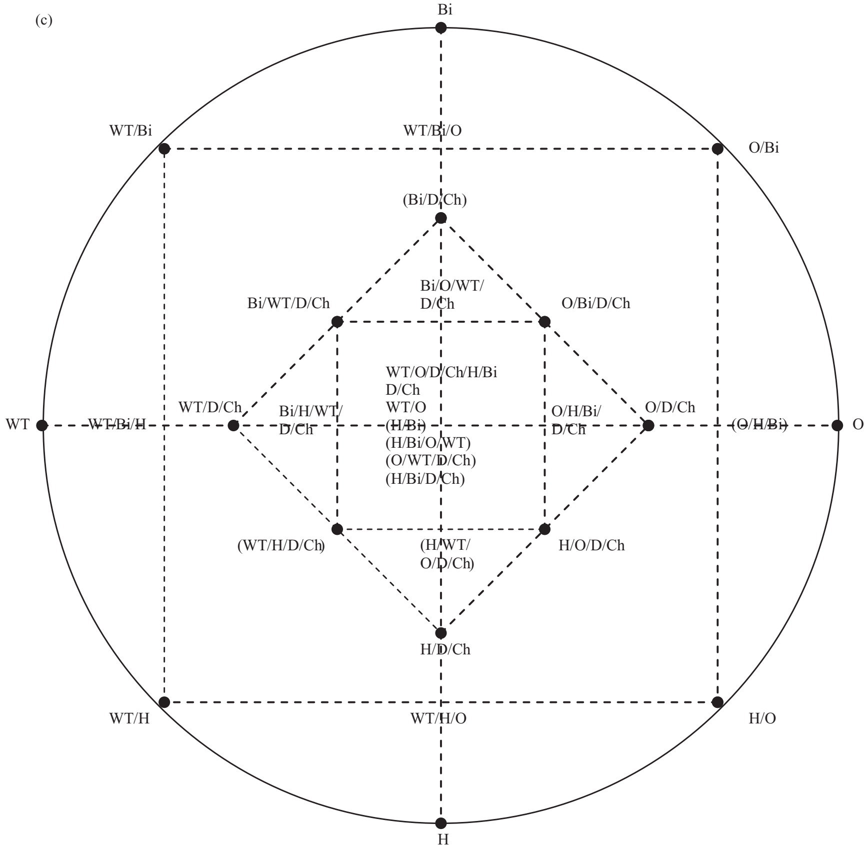


Fig. 7 Continued



Key: Bi: Bichromatic, Ch: Chromatic, D: Diatonic, H: Hexatonic, O: Octatonic, WT: Whole-Tone

along both axes. In practice, this difficulty is somewhat alleviated, as before, by the absence of some of these theoretical interfaces.

The process of genus intersection, although essentially the final part in a hierarchical process of reduction of pitch-class set information, will necessarily require one final expansion of that information involving the collation of all genus memberships (from Table 8) in order to create a catalogue (Table 11) illustrating all forms of intergeneric membership. In addition to the six categories involving just one genus, marked '1' in column 1 of Table 11 (i.e. the areas of extreme typicality and exclusivity), there are theoretically 57 other forms of intersection, nineteen of which are empty and therefore not included.[126] It can be noted that chains of contiguous set classes in any intersection row in column 3 often match lines of linkage found in the BIG chains of Tables A1–A6, although in some cases they produce different cross-generic associations. Either way, these

Table 11 Generic intersections and their member set classes

No. of genera	Generic intersection	Set classes
1	Dia node	5–23/5–35/6–Z24/6–Z25/6–Z26/ 6–32/6–33/6–Z46/6–Z47/6–Z48
1	Chr node	5–1/5–2/6–1/6–2/6–Z3/6–Z4/ 6–Z10/6–Z36/6–Z37/6–Z39
1	Hexa node	5–Z17/5–21/5–22/5–Z37/ 6–16/6–Z19/6–20/6–Z44
1	Octa node	4–Z15/4–28/4–Z29/5–31/6–Z13/ 6–Z23/6–27/6–Z28/6–Z29/6–30/ 6–Z42/6–Z45/6–Z49/6–Z50
1	Bichr node	4–6/4–8/6–Z6/6–7/6–Z38
1	WT node	5–33/6–35
2	Dia/Chr	6–8/6–9/6–Z11/6–Z40
2	Dia/Hexa	4–20/5–27/6–31
2	Dia/Octa	4–26
2	Dia/Bichr	4–23/5–14
2	Dia/WT	5–34
2	Chr/Hexa	4–7/5–3/6–15
2	Chr/Octa	4–3
2	Chr/Bichr	4–1/5–5
2	Chr/WTe	5–8
2	Hexa/Octa	4–17
2	Hexa/WT	3–12/4–19/4–24
2	Octa/Bichr	3–5/4–9/5–7/5–15/5–19/6–5/ 6–Z12/6–Z17/6–18/6–Z41/6–Z43
2	Octa/WT	6–21/6–34
2	Bichr/WT	6–22
3	Dia/Chr/Hexa	5–11/6–14
3	Dia/Chr/Octa	3–2/3–7/4–10/5–10/5–25
3	Dia/Chr/WT	3–6/4–11/4–21
3	Dia/Hexa/Octa	3–11/5–32
3	Dia/Octa/Bichr	5–29
3	Chr/Hexa/Octa	3–3/5–16
3	Chr/Octa/Bichr	5–4
3	Hexa/Octa/WT	5–26
3	Hexa/Bichr/WT	5–13/5–30
3	Octa/Bichr/WT	2–6/3–8/4–25/5–28
4	Dia/Chr/Hexa/Octa	2–3/3–10
4	Dia/Chr/Octa/Bichr	4–13/(5–Z12)/5–Z36
4	Dia/Chr/Bichr/WT	5–9/5–24
4	Dia/Hexa/Octa/Bichr	3–9/4–14/4–16/5–20/5–Z38
4	Dia/Hexa/Octa/WT	4–27
4	Chr/Hexa/Octa/Bichr	3–1/4–4/4–5/5–6/5–Z18
4	Chr/Hexa/Octa/WT	4–12
5	Dia/Chr/Hexa/Octa/Bichr	2–1/2–5/3–4/4–18
5	Dia/Chr/Hexa/Bichr/WT	4–2/4–22
5	Dia/Chr/Octa/Bichr/WT	2–2
6	Dia/Chr/Hexa/Octa/Bichr/WT	2–4

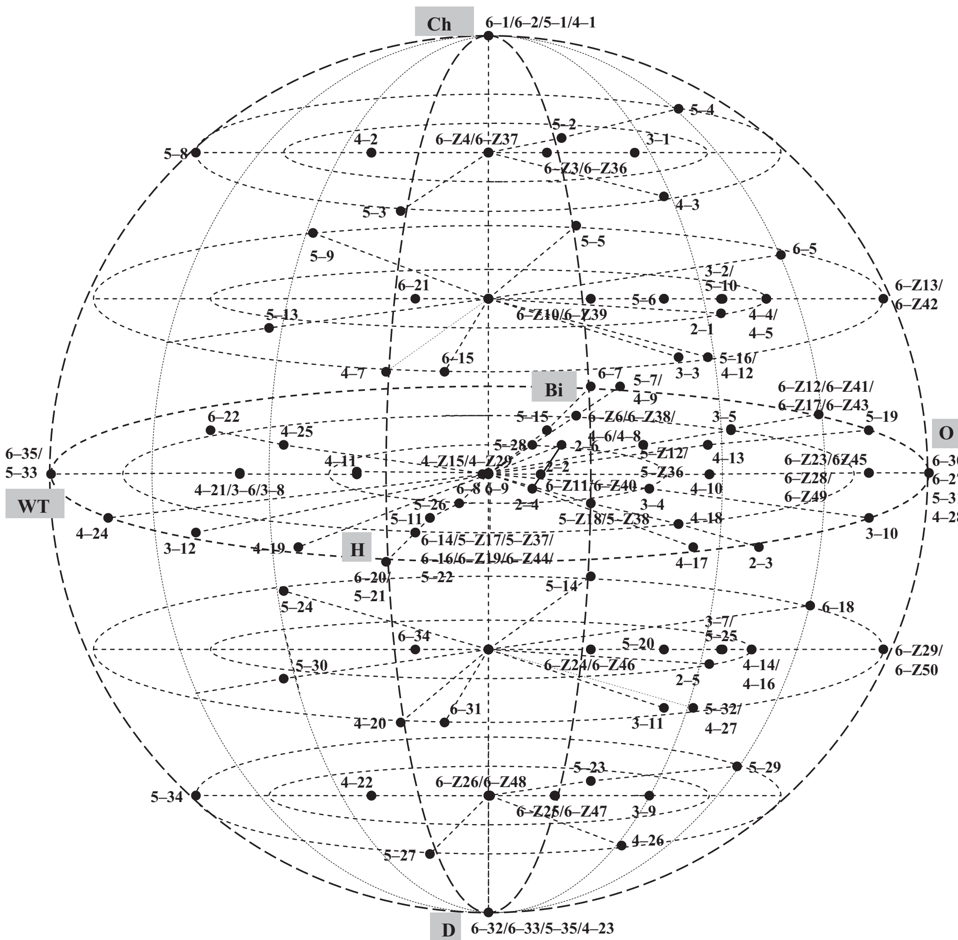
Key: Dia = Diatonic
Chr = Chromatic
Hexa = Hexatonic
Octa = Octatonic
Bichr = Bichromatic
WT = Whole-Tone

horizontal lists in column 3 pinpoint intergeneric lines of affinity. The most notable of these link the Octatonic to the Bichromatic, since these two generic areas are to some extent distinct from the other four through their common set classes. Two more features of Table 11 can be observed. Firstly, M relations are conserved, as before, within and between generic intersections. Secondly, Z-related pairs of set classes always occupy the same form of intersection and so will occupy the same point in set-class space, as they do in Quinn's Fourier balances. Although most forms of intergeneric affinity in Table 11 contain set classes of differing cardinalities, each form will nevertheless become a single general point of interface on or within the globe.

Now that the initial distribution of set classes onto points of genus intersection has been made, any more precise plotting and positioning of set classes and any 'finer distinction' of affinities relative to the prototypical hexachords or to other associated set classes will need to be facilitated in more subtle ways.[127] These more precise positions can be altered principally through reference to each set class's generic affinity, that is, its distance from each of the prototypes, as codified in Table 10, but also to its slightly different placement relative to whether it has priority as either a true subset of the genus prototype hexachord (bold type in Table 8) or as an 'exclusive' (displaying the prototypical interval-class vector patterns of Table 2 and Table 7). Fig. 8 shows all of these final set-class placements, uncluttered by other labels.[128] This final diagram has two additional horizontal planes cutting across the globe, so that the diagram can reflect all five of the intermediary levels within the Diatonic-to-Chromatic spectrum; final set-class placements must be viewed in relation to all five of these planes and the lines radiating from the central nodes of each. Once again, M-related set classes are still symmetrically arranged about a central area, the equatorial plane. It must be re-emphasised that although set-class placements are still somewhat approximate, they nevertheless show real-space affinity relations across the planes, between each of the planes and around the surface of the globe. Relative closeness or distance between set classes is illustrated by the perceived distances between them in the three-dimensional space. The illustration is therefore properly equipped to reflect affinities, to the extent that adjacent set classes nearly always register as having identical or similar affinity values when one reads horizontally across the six genus columns of Table 10. This similarity is especially noticeable between the three dyads, 2–2, 2–4 and 2–6, joined together in Fig. 8 by a line in the central disc of the equatorial plane.

In essence, Figs 6, 7 and 8 show that each of the six overlapping hemispheres (northern, southern, eastern, western, front-facing and rear-facing) represent the radiating domain of one genus, although never entirely so, demonstrating aspects of that 'radial structure' of set-class space envisioned by Quinn.[129] They also show that the horizontal planes to the north and south reflect an interface of any combination of the Hexatonic, Bichromatic, Octatonic or Whole-Tone genera with either the Chromatic or the Diatonic (but not both), and that the central, 'equatorial' plane reflects both the Chromatic-with-Diatonic interface (some-

Fig. 8 The three-dimensional set-class space



Key: Bi: Bichromatic, Ch: Chromatic, D: Diatonic, H: Hexatonic, O: Octatonic, WT: Whole-Tone

times with others as well) and the neither-Chromatic-nor-Diatonic interface (i.e. exclusively some combination of Hexatonic, Bichromatic, Octatonic or Whole-Tone).[130] Clearly, the Hexatonic, Bichromatic, Octatonic and Whole-Tone areas do not encroach on the Diatonic and Chromatic 'homeland' stretching horizontally between the north and south poles, nor do the Diatonic and Chromatic areas extend as far as the exterior equatorial rim controlled by the other four.[131]

Although the very central core of the globe is, as a matter of practical necessity, occupied by the Diatonic/Chromatic and Whole-Tone/Octatonic crossover set classes, 6–8/6–9 (0,2,3,4,5,7)/(0,1,2,3,5,7), 6–Z11/6–Z40 (0,1,2,4,5,7)/(0,1,2,3,5,8) and 6–21/6–34 (0,2,3,4,6,8)/(0,1,3,5,7,9), logically the wider central core of the three-dimensional space is also inherently occupied by those other interior set-class groupings which do not strongly display any particular genus property but which stand gregariously at the interface between several genera. These include those set classes identified as having the highest genus membership in Tables 8 (marked '+') and 11, as being central in Samplaski's histograms and as having a preponderance of neutral level '4' values in Table 10. On this basis, the most neutral set classes in generic terms also generally have the most neutral (i.e. balanced) array of interval-class vectors. These set classes are variously described as 'neutral', odourless' and 'social' by Tenkanen, as 'low-class' by Forte and as 'garbage' by Samplaski; they are further described by Forte as having 'shirttail' status.[132] It is his view that, although many of these central sets are clearly of analytical importance within the atonal repertoire, others may not belong to any genus in any clear sense and may therefore prove to be analytically unusable in terms of genus membership. We could call these sets 'weak joiners', as opposed to the more exclusively situated 'strong loners' to be found on the outer surface of the sphere. We could also say that weak joiners are weakly intrageneric but more strongly intergeneric in the way that they associate with other set classes, while strong loners are correspondingly stronger intragenerically but weaker intergenerically. We could equally say that weak joiners weakly display (i.e. balance up) those high or low interval-class vector scores characteristic of their (nominal) genera, while strong loners display them strongly. Since the weak joiners are anti-scale and anti-collectional, we might even consider them to be the most 'atonal' in quality.

So what interrelationships and particular qualities do the specific regions display, if any? In the first place, we can observe that the Whole-Tone region, uniquely, has no Z-paired set classes and has an evenly balanced but small level of interface with each of the other five genera, reflecting its equal degree of remoteness from the other five. As Tenkanen and Forte have remarked of its purely whole-tone progenitor, 'that poor guy 6–35 [(0,2,4,6,8,10)] without friends' is indeed a 'notoriously antisocial creature'.[133] By contrast, the partially related Bichromatic and Octatonic regions proffer the highest level of interface, with 33 set-class counts. Overall, each genus region displays well-connected pathways and agreeably compact structures radiating from its nodal point. Since intrageneric and intergeneric distance is measured according to a set class's level of affinity to the prototypical hexachord, the farther away, in spatial terms, a particular set class is from its prototype (displaying higher numerical values in Table 10), the more likely it is to have some degree of affinity to one or more of the other generic areas, until a point is reached when that set class is better categorised as being in a balanced (neutral) state of genus membership (displaying the value '4' in Table 10) or, ultimately, as being more closely related to one or more of the other prototypes (displaying lower numerical values in Table 10). These more distantly linked set classes, which might have been included in a particular genus under less stringent membership criteria, will actually always turn out to be correspondingly closer to other generic areas. It must be remembered, though, that a few neutral set classes, such as 5–Z12 (0,1,3,5,6), seem to demand to be considered a 'limit case' of their own because of their strong disinclination to join any genus.[134] By contrast, because ic 4 is the only interval class found in all pentachords and hexachords, set class 2–4 (0,4) is, unsurprisingly, the ultimate cross-generational set class.

Taking the wider perspective, although the global arrangement does not have lateral axial symmetry, that is, between the Hexatonic and Bichromatic and between the Whole-Tone and Octatonic areas, it does display a perfect vertical (Chromatic-to-Diatonic) symmetry, relating all sets between the two hemispheres along and around the north–south axis. This is in essence a precise corollary to the degree of symmetry exhibited between two core diatonically and chromatically oriented M_5-related collections of set classes (i.e. the M-related set classes that share an 'equal ics 2, 3, 4 and 6' feature with their M partner, labelled 'a', 'b', etc. in Table 9). A contrasting third set-class collection is located across the whole of the intervening horizontal equatorial plane and comprises those more 'atonal' singleton set classes, each of which displays the same number of ics 1 and 5; this collection demonstrates no bias towards either the Diatonic or the Chromatic (i.e. the M-invariant set classes in Table 9).[135] Table 12 displays twelve 'special' set classes (six Z-related pairs) which are cross-relational, exhibiting both the 'sharing ics 2, 3, 4 and 6 with an M-related partner' property and the 'equal ics 1 and 5' property normally held only by M invariants. Five additional set classes, 4–11 (0,1,3,5), 5–11 (0,2,3,4,7), 5–Z12 (0,1,3,5,6), 5–Z36 (0,1,2,4,7) and 6–8 (0,2,3,4,5,7) (and their complements), are also cross-relational, in the sense that each acts as an intervallic intermediary between those Diatonic-versus-Chromatic pairs most similar to them (and their complements). Table 13 shows how these five intermediaries have midway or averaged ic 1 and ic 5 ratings compared to the ic 1 and ic 5 content being swapped by the associated M-related pair, but at the same time have the exact same ratings for ics 2, 3, 4 and 6 as the associated M-related pair. It will hardly come as a surprise that this privileged group of seventeen cross-relational set classes correlates in large part with those set classes already identified as being 'neutral'. Together, they stand at some kind of central proto-generic atonal-diatonic-chromatic interface. Once again, several of Quinn's desiderata have been confirmed by these wider issues, including those of distance and similarity (key criteria 1, 2 and 4), intrageneric and intergeneric affinity (key criterion 5), the properties of interval content, subset structure and transformational symmetry (key criterion 7) and the placement of prototypes (key criterion 8).

This pitch-class set space odyssey has explored many of the highways and byways of set-class theory since its starting point. We could say that Hanson was pioneering a process by lighting the way towards a taxonomy of the set-class universe. Others, most notably Quinn, have subsequently contributed to this possibility by clarifying the direction that this path needs to take and by bringing together concepts such as set-class and interval-class (similarity) relations and

Table 12 The twelve M-related (and Z-related) set classes sharing a complete interval-class vector with their partner

		Interval-class vector
4–Z15 (0,1,4,6)	4–Z29 (0,1,3,7)	[111111]
5–Z17 (0,1,3,4,8)	5–Z37 (0,3,4,5,8)	[212320]
5–Z18 (0,1,4,5,7)	5–Z38 (0,1,2,5,8)	[212221]
6–Z6 (0,1,2,5,6,7)	6–Z38 (0,1,2,3,7,8)	[421242]
6–Z19 (0,1,3,4,7,8)	6–Z44 (0,1,2,5,6,9)	[313431]
6–Z11 (0,1,2,4,5,7)	6–Z40 (0,1,2,3,5,8)	[333231]

Table 13 Five 'intermediate' set classes situated intervallically between Diatonic and Chromatic M-related set classes (and vertically in Fig. 8)

	Set class	ic vector	Set class	ic vector	Set class	ic vector	Set class	ic vector
Chromatic set class	4–2 (0,1,2,4)	[**2**211**0**0]	5–3 (0,1,2,4,5)	[**3**222**1**0]	5–4 (0,1,2,3,6)	[**3**221**1**1]	6–1 (0,1,2,3,4,5)	[**5**432**1**0]
Intermediate set class	4–11 (0,1,3,5)	[**1**211**1**0]	5–11 (0,2,3,4,7)	[**2**222**2**0]	5–Z12 (0,1,3,5,6) 5–Z36 (0,1,2,4,7)	[**2**221**2**1]	6–8 (0,2,3,4,5,7)	[**3**432**3**0]
Diatonic set class	4–22 (0,2,4,7)	[**0**211**2**0]	5–27 (0,1,3,5,8)	[**1**222**3**0]	5–29 (0,1,3,6,8)	[**1**221**3**1]	6–32 (0,2,4,5,7,9)	[**1**432**5**0]

Note: Incrementally changing ics 1 and 5 are shown vertically in bold.

cyclic and multiplicative processes. In this scheme of things, I would like to think that my contribution to the ongoing discourse, by delving into several new issues, represents a step forward along the path to understanding set-class relationships. A new methodology for the creation of a comprehensive array of BIG chains has led to the proposition of a sixfold system of genera. The formation of a diatonic-to-chromatic, M_5-induced spectrum of symmetrically arranged set classes, together with affinity scalings derived from genus-based interval-class characteristics, have in turn led to the construction of a metaphorical representation of that system of genera in three-dimensional space. I hope that these ideas might stimulate further interest in this field and suggest that the four perspectives on set-class affinity presented here, namely the BIG chains, the system of genera, the interval-class scalings and the single-page three-dimensional representation of the genera, could offer particularly fruitful starting points as analytical tools in the future. Whatever direction this might take, I hope that my contribution will result in further explorations into intergeneric set-class space.

NOTES

1. Allen Forte, 'Pitch-Class Set Genera and the Origin of Modern Harmonic Species', *Journal of Music Theory*, 32/ii (1988), pp. 187–270.

2. Tore Eriksson, 'The IC Max Point Structure, MM Vectors and Regions', *Journal of Music Theory*, 30 (1986), pp. 95–111; and Richard S. Parks, *The Music of Claude Debussy* (New Haven, CT: Yale University Press, 1989), and 'Pitch-Class Set Genera: My Theory, Forte's Theory', *Music Analysis*, 17/ii (1998), pp. 206–26.

3. Forte, 'Pitch-Class Set Genera', p. 218.

4. *Ibid.*, pp. 187, 204, 211 and 230. An alternative diatonic-to-chromatic interpretation of Forte's spectrum of genera will be posited later in this article.

5. Parks, 'Pitch-Class Set Genera: My Theory, Forte's Theory', p. 213. Parks's genera are analytically and contextually, rather than theoretically, generated from the 'cynosural' set class(es) of a piece or repertoire. Allen Forte, 'Afterword', *Music Analysis*, 17/ii (1998), p. 243.

6. This was at the Cambridge University Music Analysis Conference Round Table on pc set genera, 8 August 1997, subsequently formalised in Allen Forte, 'Round Table: Response and Discussion', *Music Analysis*, 17/ii (1998), pp. 227–36.

7. Parks's genera, although numbering only five in his Debussy study, could potentially extend to 'in terms of forty' or possibly several thousand and run into the same problem of large membership; see Parks, *The Music of Claude Debussy*; Forte, 'Round Table', p. 230; and Parks, 'Pitch-Class Set Genera: My Theory, Forte's Theory', p. 210.

8. Ian Quinn, 'General Equal-Tempered Harmony (Introduction and Part 1)', *Perspectives of New Music*, 44/ii (2006), p. 141; see also Quinn, 'Listening to Similarity Relations', *Perspectives of New Music*, 39/ii (2001), p. 153.

9. Bernard Gates, 'The Codification of Pitch Organisation in the Early Atonal Works of Alban Berg' (PhD diss., Open University, 1999), pp. 161–2 and 182–5, tables 5.6 (erroneously labelled 'table 2'), 6.3 and 6.4.

10. Robert Morris, 'A Similarity Index for Pitch-Class Sets', *Perspectives of New Music*, 18/i–ii (1979–80), pp. 445–60; Eric J. Isaacson, 'Similarity of Interval-Class Content between Pitch-Class Sets: the IcVSIM Relation', *Journal of Music Theory*, 34/i (1990), pp. 1–28; Michael Buchler, 'Relative Saturation of Interval and Set Classes: A New Model for Understanding PcSet Complementation and Resemblance', *Journal of Music Theory*, 45/ii

(2001), pp. 263–343; David Lewin, 'A Response to a Response: On PcSet Relatedness', *Perspectives of New Music*, 18/i–ii (1979–80), pp. 498–502; John Rahn, 'Relating Sets', *Perspectives of New Music*, 18/i–ii (1979–80), pp. 483–98; and Marcus Castrén, 'RECREL: a Similarity Measure for Set-Classes' (PhD diss., Sibelius Academy, 1994).

11. Quinn, 'Listening to Similarity Relations', pp. 141, 151 and 153; Quinn, 'General Equal-Tempered Harmony (Introduction and Part 1)', pp. 142–4; Thomas R. Demske, 'Relating Sets: On Considering a Computational Model of Similarity Analysis', *Music Theory Online*, 1/ii (1995); and Art Samplaski, 'Mapping the Geometries of Pitch-Class Set Similarity Measures via Multidimensional Scaling', *Music Theory Online*, 11/ii (2005).

12. Quinn, 'General Equal-Tempered Harmony (Introduction and Part 1)', p. 135; and Samplaski, 'Mapping the Geometries', paras 8–18.

13. Quinn, 'General Equal-Tempered Harmony (Introduction and Part 1)', pp. 134–5; and Quinn, 'Listening to Similarity Relations', pp. 134–6.

14. Quinn, 'Listening to Similarity Relations', pp. 146–9 and 151–3; and Samplaski, 'Mapping the Geometries', para. 19.

15. Joseph N. Straus, 'Voice Leading in Set-Class Space', *Journal of Music Theory*, 49/i (2005), pp. 45–108; Dmitri Tymoczko, 'The Geometry of Musical Chords', *Science*, 313/5783 (2006), pp. 72–4; and Clifton Callender, Ian Quinn and Dmitri Tymoczko, 'Generalized Voice-Leading Spaces', *Science*, 320/5874 (2008), pp. 346–8.

16. Callender, Quinn and Tymoczko state that although '[m]usically, … we can use voice-leading size to measure distance between chord-types' ('Supporting Online Material for "Generalized Voice-Leading Spaces" ', p. 6), they nevertheless 'abandon the idea that voice-leading size corresponds to something like distance in the quotient space' (p. 7); see also their note S8 on p. 49.

17. This is an issue which Hoffman seeks to address in his study of the relationship between voice leading and Fourier spaces; see Justin Hoffman, 'On Pitch-Class Set Cartography: Relations between Voice-Leading Spaces and Fourier Spaces', *Journal of Music Theory*, 52/ii (2008), pp. 219–49.

18. Quinn points out the fundamental difference between intervallic/inclusional similarity and voice-led similarity and seems to imply that a commitment to one type of similarity necessarily 'takes the place of' the other 'completely different' commitment; see Quinn, 'General Equal-Tempered Harmony (Introduction and Part 1)', p. 119, and 'General Equal-Tempered Harmony: Parts 2 and 3', *Perspectives of New Music*, 45/i (2007), p. 61.

19. Straus, ‘Voice Leading in Set-Class Space’, pp. 51–2, 56, 60 and 62–3; and Quinn, ‘General Equal-Tempered Harmony: Parts 2 and 3’, *Perspectives of New Music*, 45/i (2007), p. 47, ex. 43.

20. See n. 9 for the relevant references within my thesis; the term ‘bichromatic’ will be defined a little later.

21. These chains were originally called ‘generative sequence-related chains of equivalence/similarity’ in table 6.3 of my thesis. Such IG chains are implicit in Hanson’s interval-class projections and are made explicit in Eriksson’s series of set types; see Eriksson, ‘The IC Max Point Structure’, p. 97, ex. 2.

22. Eriksson, ‘The IC Max Point Structure’, pp. 95–6; and Quinn, ‘General Equal-Tempered Harmony (Introduction and Part 1)’, p. 127.

23. This is in contrast to those more precise unitary and measured illustrations of affinity, such as Quinn’s Fourier balances, which have been less useful in comparing set classes of different cardinalities: see Quinn, ‘General Equal-Tempered Harmony: Parts 2 and 3’, pp. 47–8; and Hoffman, ‘On Pitch-Class Set Cartography’, p. 226. The problem is sidestepped here by taking an intersection-based approach to affinity.

24. Quinn returns repeatedly to these issues during the presentation of his landmark article: Quinn, ‘General Equal-Tempered Harmony (Introduction and Part 1)’, pp. 116, 122, 124–6 and 134–6, and ‘General Equal-Tempered Harmony: Parts 2 and 3’, pp. 19, 45 and 61.

25. Quinn, ‘General Equal-Tempered Harmony (Introduction and Part 1)’, pp. 115–21.

26. Howard Hanson, *Harmonic Materials of Modern Music: Resources of the Tempered Scale* (New York: Appleton-Century-Crofts, 1960); Eriksson, ‘The IC Max Point Structure’; Forte, ‘Pitch-Class Set Genera’; Buchler, ‘Relative Saturation of Interval and Set Classes’; Ian Quinn, ‘A Unified Theory of Chord Quality in Equal Temperaments’ (PhD diss., Eastman School of Music, University of Rochester, 2004), p. 23; and Hoffman, ‘On Pitch-Class Set Cartography’. Quinn has more recently rescinded his view that the six interval classes have a direct association with the ‘six evident qualitative genera’; Quinn, ‘General Equal-Tempered Harmony (Introduction and Part 1)’, p. 121.

27. Eriksson, ‘The IC Max Point Structure’, pp. 97 and 99, exs 2 and 3. Eriksson points out that the whole-tone scale, set class 6–35 (0,2,4,6,8,10), although central to Hanson’s ic 2 (0,2) projection and his own region 2, also encompasses ics 4 and 6, and that the octatonic scale, set class 8–28 (0,1,3,4,6,7,9,10), representative of Hanson’s ic 3 (0,3)

projection and his own region 3, similarly incorporates ic 6. Quinn agrees that the octatonic scale is just as much generated from ic 6 as it is from ic 3; see Quinn, 'General Equal-Tempered Harmony: Parts 2 and 3', p. 24. Although ic 4 purportedly generates the hexatonic region or genus, it is the most profligate of the interval classes, being the only interval class not to score zero among all pentachords and hexachords; for this reason, it might be considered the least able of the interval classes to unilaterally initiate or exemplify a distinct genus or region. It is clear that this kind of diversity also extends to the incorporation of set classes of cardinals 3 and 4 as well. For instance, while the augmented triad, set class 3–12 (0,4,8), has high ic 4 content, and is confined by Hanson to his ic 4 (0,4), i.e. hexatonic, projection, it is also a quality shared by the whole-tone scale (0,2,4,6,8,10) and therefore appears in two of Eriksson's regions, 4 and 2. Meanwhile, set class 4–9 (0,1,6,7), confined by Hanson to his ic 6 (0,6) projection, is also found within the octatonic scale (0,1,3,4,6,7,9,10) and therefore also appears in two of Eriksson's regions, 6 and 3. Similarly, set classes 3–8 (0,2,6) and 4–25 (0,2,6,8), both typical of Hanson's ic 2 (0,2), i.e. whole-tone, category, having high ics 2, 4 and 6 (although not actually part of his ic 2 (0,2) projection), are also shared by collections representative of two other projections, set class 6–7 (0,1,2,6,7,8) from the ic 6 and the octatonic scale (0,1,3,4,6,7,9,10) from the ic 3, and therefore appear in three of Eriksson's regions, 6, 2 and 3. Finally, set class 4–19 (0,1,4,8) leans towards the whole-tone as well as the hexatonic and therefore appears in Eriksson's regions 2 and 4; see Eriksson, 'The IC Max Point Structure', pp. 108–9.

28. Quinn, 'General Equal-Tempered Harmony (Introduction and Part 1)', p. 130, and 'General Equal-Tempered Harmony: Parts 2 and 3', pp. 11–12, 22 and 28–9.

29. Maximally even versus maximally compact situations inform Quinn's maximally even subgenera and F(12, 1) prototypicality measures, Straus's voice-leading spaces for each cardinality, Callender, Quinn and Tymoczko's generalised voice leading spaces and Hoffman's displacement spaces; see Quinn, 'General Equal-Tempered Harmony: Parts 2 and 3', pp. 5, 12, 26 and 46–7, and 'General Equal-Tempered Harmony (Introduction and Part 1)', p. 121; Straus, 'Voice Leading in Set-Class Space', pp. 67–71; Callender, Quinn and Tymoczko, 'Generalized Voice-Leading Spaces'; and Hoffman, 'On Pitch-Class Set Cartography', p. 221.

30. Quinn, 'Listening to Similarity Relations', pp. 132–3 and 154.

31. Eriksson, 'The IC Max Point Structure', pp. 96–7; Buchler, 'Relative Saturation of Interval and Set Classes', pp. 271–3; and Hanson, *Harmonic Materials of Modern Music*, p. 28. Hanson's development of his six categories from these basic chains involves the incremental accumulation of

one 'foreign tone' at a time into the projection, a fifth (or semitone) above the starting point. See also Quinn, 'General Equal-Tempered Harmony (Introduction and Part 1)', p. 128.

32. Quinn, 'General Equal-Tempered Harmony (Introduction and Part 1)', p. 131.

33. I have taken these category characteristics from *ibid.*, p. 126.

34. Hanson, *Harmonic Materials of Modern Music*, as illustrated in Quinn, 'General Equal-Tempered Harmony (Introduction and Part 1)', p. 130, ex. 2; Eriksson, 'The IC Max Point Structure', p. 96; and Buchler, 'Relative Saturation of Interval and Set Classes', pp. 271–3.

35. 'Distinct', 'accessible', 'familiar' and 'instinctive' are terms used by Quinn, Michael Russ and others: see Quinn, 'General Equal-Tempered Harmony (Introduction and Part 1)', pp. 123, 131 and 142; and Forte, 'Round Table', p. 235.

36. 'Bichromatic' also conveniently suggests opposite colours, as on a colour wheel: this genus could equally be called 'bidiatonic', since set class 6–7 can alternatively be partitioned into two (transpositionally combined) 'cycle of fifths' segments a tritone apart, or three ic 5–spaced tritones, i.e. (0,7,2)/(6,1,8) or (0,6)/(7,1)/(2/8). This genus was originally called '6-7' in my thesis; see Gates, 'The Codification of Pitch Organisation in Early Atonal Berg', table 6.4. A third partitioning of set class 6–7 would give inversionally combined instances of set class 3–5, i.e. (0,1,6)/(2,7,8). Although Samplaski does not consider this (or any other) hexachord as a prototype, he does term this dimension '016-saturation' or 'pitch-classes clumped at opposite sides of the chroma wheel'; see Samplaski, 'Mapping the Geometries', para. 40.

37. Set class 6–30, the *Petrushka* chord, is one of Eriksson's 'maxpoints'. It also 'contains' multiple instances of the two all-interval tetrachords, 4–Z15 (0,1,4,6) and 4–Z29 (0,1,3,7), i.e. as (**0**,**1**,3,6,**7**,**9**) or (0,**1**,**3**,**6**,7,9) in the case of 4–Z15, and as (**0**,**1**,**3**,6,7,9) or (0,**1**,3,**6**,**7**,**9**) in the case of 4–Z29.

38. The hexatonic prototype, 6–20, has the lowest ic 2 content of all hexachords (uniquely zero) *and* the highest ic 4 content (six), the latter equalled only by 6–35 (*and* it shares the lowest ic 6 content [zero] with four other hexachords). Although Hanson's set class 6–27 uniquely has five instances of the octatonically inclined ic 3, our chosen octatonic prototype, 6–30, has the second-highest ic 3 content (four) *but also* the highest ic 6 content (three), the latter equalled only by 6–7 and 6–35. The whole-tone prototype, 6–35, uniquely has the highest possible counts for ics 2, 4 and 6 (six, six and three respectively) *and* the lowest possible counts for ics 1, 3 and 5 (zero in each case). The diatonic prototype, 6–32,

uniquely has the highest ic 5 content of all hexachords (five), the lowest 6 (zero) *and* the second lowest ic 1 content (one), the latter bettered only by 6–35 and equalled only by 6–33 and 6–34. The chromatic prototype, 6–1, the converse of the diatonic 6–32, uniquely has the highest ic 1 content of all hexachords (five), the lowest ic 6 (zero) *and* the second-lowest ic 5 content (one), the latter bettered only by 6–35 and equalled only by 6–2 and 6–21. Finally, the bichromatic prototype, 6–7, is the only hexachord to have the second-highest ic 1 content (four), bettered only by 6–1, *and* the second-highest ic 5 content (four), bettered only by 6–32, *and* the lowest ic 3 content (zero), equalled only by 6–35, *and* the highest ic 6 content (three), equalled only by 6–30 and 6–35.

39. Atte Tenkanen, 'A Linear Algebraic Approach to Pitch-Class Set Genera', in Timour Klouche and Thomas Noll (eds), *Mathematics and Computation in Music: First International Conference, MCM 2007, Berlin, Germany, May 18–20, 2007, Revised Selected Papers*, Communications in Computer and Information Science 37 (Berlin, Heidelberg and New York: Springer, 2009), p. 527; I. T. Joliffe, *Principal Component Analysis*, 2nd edn (Berlin, Heidelberg and New York, 2002); Eriksson, 'The IC Max Point Structure', p. 97 (Eriksson's other maxpoints are set classes 2–2 (0,2), 2–3 (0,3), 2–4 (0,4), 2–6 (0,6), 3–5 (0,1,6), 3–6 (0,2,4), 3–8 (0,2,6), 3–10 (0,3,6), 3–12 (0,4,8), 4–9 (0,1,6,7), 4–21 (0,2,4,6), 4–24 (0,2,4,8), 4–25 (0,2,6,8), 4–28 (0,3,6,9), 5–7 (0,1,2,6,7), 5–19 (0,1,3,6,7), 5–21 (0,1,4,5,8), 5–28 (0,2,3,6,8), 5–31 (0,1,3,6,9) and 5–33 (0,2,4,6,8), all of which have been included in the growth chains already quoted); Buchler, 'Relative Saturation of Interval and Set Classes', pp. 271–3; and Quinn, 'General Equal-Tempered Harmony: Parts 2 and 3', pp. 27–9.

40. Quinn, 'General Equal-Tempered Harmony: Parts 2 and 3', p. 30, and 'General Equal-Tempered Harmony (Introduction and Part 1)', pp. 123 and 136; Eriksson, 'The IC Max Point Structure', p. 104 and p. 107, ex. 11; and Tenkanen, 'A Linear Algebraic Approach', p. 525. M structure is a relation that Eriksson has derived from Lord's cardinality-based maximum similarity relation: see Charles H. Lord, 'Intervallic Similarity Relations in Atonal Set Analysis', *Journal of Music Theory*, 25/i (1981), pp. 91–111.

41. Forte, 'Pitch-Class Set Genera', p. 201.

42. Each can nevertheless be expressed in four mathematical forms, commensurate with octave transpositions within an ordered sequence of pitch classes. By way of illustration, the OTIC (1,1) (11,11), as represented by the ordered sequence C–C♯–D or its retrograde, might be articulated notationally as C4–C♯4–D4 (up interval-1, up interval-1, i.e. 1 + 1 = 2, mod. 12), as C4–C♯4–D3 (up interval-1, down interval-11, i.e. 1 − 11 = 2, mod. 12), as D4–C♯5–C6 (up interval-11, up interval-11, i.e. 11 + 11 = 10,

the inverse of 2, mod. 12) or as D4–C♯5–C5 (up interval-11, down interval-1, i.e. 11 – 1 = 10, the inverse of 2, mod. 12).

43. Combination cycles were first investigated by Lambert, who lists 132 (11 × 12) combination cycles, which he then reduces to 71 T_nI combination cycles; see J. Philip Lambert, 'Interval Cycles as Compositional Resources in the Music of Charles Ives', *Music Theory Spectrum*, 12/i (1990), pp. 56–7. These 71 combination cycles can be further reduced to 60 by eliminating those that involve zero in each of his T_nI rows; finally, these 60 T_nI forms can be halved to my 30 by combining the pairs of numerals (or singles that represent pairs of the same numeral) that sum the same way, modulo 12, from each of ($T_1I/T_{11}I$), ($T_2I/T_{10}I$), (T_3I/T_9I), ($T_4I/T_8/I$), (T_5I/T_7I) and (T_6I), e.g. the pairs (1,1) (11,11) count as one cell, and as one basic combination cycle, since they sum to 2 or 10, modulo 12.

44. All sums in Table 3 follow the patterns below:
±11/1 for cells generated from interval-1 combination cycles
±10/2 for cells generated from interval-2 combination cycles
±9/3 for cells generated from interval-3 combination cycles
±8/4 for cells generated from interval-4 combination cycles
±7/5 for cells generated from interval-5 combination cycles
±6 for cells generated from interval-6 combination cycles

For each column in Table 3, the total within each trichordal cell (e.g. 1 + 1 or 11 + 11, mod 12) equals the interval-class number of the combination cycle's initiating cycle, or its mod 12 inversion (e.g. 2 or 10); conversely, for each column, the difference between either the first element of each pair of intervals or the second element of each pair of intervals within each trichordal cell (e.g. in the same case, 1 - 11 or 11 - 1, mod 12), also equals the interval-class number of the combination cycle's initiating cycle, or its mod 12 inversion.

45. This was originally table 6.1 in my thesis, where it was also set out notationally as ex. 6.2.

46. The M_5 and M_7 relationships between combination cycle cells in Bartók, investigated by Gollin, and the I-chain mutual relationships within motivic chains, also in Bartók, published more or less concurrently by Straus, are both different to the cell relationships shown in Tables 3 and 4 but, remarkably, are essentially the same as each other: in both cases, cells from *different* combination cycles are conjoined or correlated, e.g. in Gollin's ex. 6, the interval-1 combination cycle cell (6,7) (6,5) is linked to the interval-5 combination cycle cell (6,1) (6,11), both of which form set class 3–5 (0,1,6), while similarly, in Straus's ex. 2c, the interval-1 combination cycle cell (2,9) (3,10), spelled <+2, –3> or <–2, +3> in Straus's terminology, forming set class 3–2 (0,1,3), is linked to the interval-5

combination cycle cell (2,3) (9,10), spelled <–3, –2>, forming set class 3–7 (0,2,5). There are 20 Gollin M cycles in all; ten of them correspond to Straus's ten i-chain pairs (reflecting 20 of the combination cycles) and ten are self-replicating (the other ten combination cycles). See Edward Gollin, 'Multi-Aggregate Cycles and Multi-Aggregate Serial Techniques in the Music of Béla Bartók', *Music Theory Spectrum*, 29/ii (2007), p. 149; and Joseph N. Straus, 'Motivic Chains in Bartók's Third String Quartet', *Twentieth-Century Music*, 5/i (2008), p. 28.

47. Eriksson's ex. 2 is also set out in the form of IG chains; see Eriksson, 'The IC Max Point Structure', p. 97.

48. In this and all subsequent respects, I am taking the pragmatic line that seven-, eight-, nine- and ten-element set classes will display exactly the same characteristics and interrelationships as their complements of cardinals 2–5. Consequently, these larger set classes will normally be implied through their complementary association but omitted for reasons of space and clarity.

49. There is an expansion of the bichromatic system to include 6–Z6 (0,1,2,5,6,7) and its Z-partner 6–Z38 (0,1,2,3,7,8), both of which are supersets of 6–7's principal subset, 5–7 (0,1,2,6,7); there is also an expansion of the octatonic to include 6–Z13 (0,1,3,4,6,7), 6–Z23 (0,2,3,5,6,8), 6–Z49 (0,1,3,4,7,9) and 6–Z50 (0,1,4,6,7,9), all of which are subsets of set class 7–31 (0,1,3,4,6,7,9) and therefore of 8–28 (0,1,3,4,6,7,9,10), the octatonic scale; further, there is an expansion of the diatonic to include 6–8 (0,2,3,4,5,7) and 6–Z26 (0,1,3,5,7,8), the latter being a subset of 7–35 (0,1,3,5,6,8,10), the diatonic scale; finally, there is an expansion of the chromatic to include 6–8 (0,2,3,4,5,7) and 6–Z4 (0,1,2,4,5,6), the latter being a subset of 7–1 (0,1,2,3,4,5,6), the chromatic heptachord. It can be noted here that set class 6–8 (0,2,3,4,5,7), the other all-combinatorial hexachord that has sometimes been included as representative of a genus, e.g. by Eriksson in his overlapping diatonic/chromatic region 7, is the superset both of 5–2 (0,1,2,3,5), itself a subset of the chromatic prototype 6–1 (0,1,2,3,4,5), and of 5–23 (0,2,3,5,7), itself a subset of the diatonic prototype 6–32 (0,2,4,5,7,9); this crossover area between chromatic and diatonic will be investigated at length later.

50. Cohn makes reference to 'minimal perturbations of a symmetrical division of the octave'; see Richard Cohn, 'Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions', *Music Analysis*, 15/i (1996), p. 39, n. 40.

51. Hoffman considers the effect of intervallic displacements of varying sizes in his study of Fourier spaces: see Hoffman, 'On Pitch-Class Set Cartography', p. 231, table 1.

52. These foundational pentachords are 5–21 (0,1,4,5,8) for the hexatonic, 5–7 (0,1,2,6,7) and 5–15 (0,1,2,6,8) for the bichromatic, 5–19 (0,1,3,6,7), 5–28 (0,2,3,6,8) and 5–31 (0,1,3,6,9) for the octatonic, 5–33 (0,2,4,6,8) for the whole-tone, 5–23 (0,2,3,5,7), 5–27 (0,1,3,5,8) and 5–35 (0,2,4,7,9) for the diatonic, and 5–1 (0,1,2,3,4), 5–2 (0,1,2,3,5) and 5–3 (0,1,2,4,5) for the chromatic. The hexachord families of Table 6 were first explained and set out in my thesis, p. 161 and p. 204, table 5.6 (where it was erroneously labelled 'table 2').

53. Eriksson, 'The IC Max Point Structure', p. 107, ex. 11; and Tenkanen, 'A Linear Algebraic Approach', p. 528, fig. 3.

54. Eriksson, 'The IC Max Point Structure', pp. 101–2, exs 4 and 6.

55. Buchler, 'Relative Saturation of Interval and Set Classes', p. 274, fig. 5.

56. Eriksson, 'The IC Max Point Structure', pp. 102–3, ex. 7; and Quinn, 'Listening to Similarity Relations', pp. 128, 131, 148 and 152, exs 9, 17 and 18.

57. This compounding of relationships would ultimately cause the rise of an 'effectively inexhaustible sea' such as is found in the systems of Alois Hába and Howard Hanson, producing a 'staggeringly complex' network of relationships requiring 'one monstrous graph'; see Rahn, 'Relating Sets', pp. 494 and 496–7; and Jonathan W. Bernard, 'Chord, Collection, and Set in Twentieth-Century Theory', in James M. Baker, David W. Beach and Jonathan W. Bernard (eds), *Music Theory in Concept and Practice* (Rochester, NY: University of Rochester Press, 1997), pp. 29 and 48–9.

58. Bonded pairs of set classes of the same cardinality within BIG chains have the same kind of close kinship as that created through interval-class displacement in the hexachord families of Table 6, and in the spaces suggested in Hoffman, 'On Pitch-Class Set Cartography'.

59. Tables A1–A6 represent somewhat modified and expanded versions of table 6.3 in my thesis, pp. 207–12.

60. A few hybrid BIG chains, transgenerational in their progression from smaller to larger sets, have had to be excluded from Tables A1–A6, given the prevailing desire at this stage to formulate well-defined and distinctive genera.

61. From this point in the article, the genera presented in Table 8 will be referred to with an initial capital (e.g. Diatonic), while more general references will retain the initial lower case. Complements of set classes have been omitted from Table 8 for reasons of space but can be assumed to be implicitly present alongside the quoted membership. Eriksson's set classes associated with each genus under interval-class vector counts are

now confirmed in Table 8. The genera shown here were originally presented in slightly different form as table 6.4 in my thesis, pp. 213–4.

62. Samplaski's comments and oppositional histograms confirm my findings. He states that '[t]he notion of different kinds of pcsets at the extremes of the dimensions invites comparison with various models of pcset families, or genera': Samplaski, 'Mapping the Geometries', para. 54. Some of his histograms reveal strong generic associations, i.e. Bichromatic set classes opposed to Octatonic ones in his fig. 8, Hexatonic set classes opposed to Diatonic/Chromatic ones in his fig. 14, and Hexatonic and Bichromatic set classes opposed to Octatonic ones in his fig. 15. Others have associations that are less strong, i.e. Whole-Tone set classes opposed to ones from the Bichromatic and Octatonic in his fig. 13, Hexatonic set classes opposed to Diatonic/Chromatic ones in his fig. 10, and Diatonic/Chromatic set classes opposed to ones from the Bichromatic and Octatonic in his fig. 16.

63. This deviancy may be explained by the fact that 5–Z12 (0,1,3,5,6) [222121] has only half the number of distinct forms of its Z-partner, 5–Z36 (0,1,2,4,7) [222121], owing to its symmetry, a quality unique among Z-designated set classes, and to the fact that it is not contained within its complement, 7–Z12 (0,1,2,3,4,7,9) [444342], a lack of relationship which is also unique among complementary set classes.

64. 4–Z15 and 4–Z29 insinuate themselves into Eriksson's whole-tone and bichromatic regions as well, and in Quinn's lists they also subsist in his bichromatic area; see Eriksson, 'The IC Max Point Structure', p. 102; and Quinn, 'Listening to Similarity Relations', p. 148.

65. In fact, the supersets of *every* Z-designated set class are the complements of the subsets of its Z-partner, including the single tetrachord pair, 4–Z15/4–Z29 (0,1,4,6)/(0,1,3,7) within an octatonic (8–28 (0,1,3,4,6,7,9,10)) aggregate, and the three pentachord pairs, 5–Z12/5–Z36 (0,1,3,5,6)/(0,1,2,4,7), 5–Z17/5–Z37 (0,1,3,4,8)/(0,3,4,5,8) and 5–Z18/5–Z38 (0,1,4,5,7)/(0,1,2,5,8) within ten-element aggregates. It can be asserted here that the various types of complementation displayed by the BIG chains in Tables A1–A6 emphatically demonstrate complementation as a valid form of equivalence, rather than instinctively assuming it, as does Forte in his rule 1 for genus formation; see Forte, 'Pitch-Class Set Genera', p. 192.

66. This equivalence will be discussed at length later.

67. Two BIG chains terminating with set class 6–14 are found in Tables A1 (hexatonic), A5 (diatonic) and A6 (chromatic); three BIG chains incorporating set classes 6–5 and 6–18 and two terminating with set class

6–Z43 are found in Tables A2 (bichromatic) and A3 (octatonic); and a single BIG chain terminating with set class 6–9 is found in Tables A5 (diatonic) and A6 (chromatic).

68. Eriksson, 'The IC Max Point Structure', pp. 102–3, ex. 7.

69. That is, the Bichromatic with Forte's genera 1/2, the Octatonic with his genus 3, the Chromatic with his genera 5/6, the Hexatonic with his genera 8/9/10 and the Diatonic with his genera 11/12; meanwhile, Forte's genus 4 has elements in common with both my Hexatonic and Whole-Tone genera.

70. The higher incidence of dyads and trichords within a genus correlates largely with their level of subset-of-hexachord-prototype status within that genus.

71. Quinn discusses 'characteristic' set classes that intuitively fill a harmonic space in 'General Equal-Tempered Harmony: Parts 2 and 3', p. 51.

72. Parks, 'Pitch-Class Set Genera: My Theory, Forte's Theory', pp. 207 and 209; and Forte, 'Pitch-Class Set Genera', pp. 192 and 234–5.

73. This range of membership is not so unexpected when we consider that the interval-class factor which binds the Hexatonic, Diatonic and Chromatic prototypes is zero ic 6, and that this is the *only* distinctive feature in set class 5–11's (0,2,3,4,7) interval-class vector [222220].

74. Allen Forte, *The Structure of Atonal Music* (New Haven, CT: Yale University Press, 1973), pp. 167–9.

75. Eric J. Isaacson, 'Issues in the Study of Similarity in Atonal Music', *Music Theory Online*, 2/vii (1996), para. 22.

76. *Ibid.*, para. 27.

77. Forte, *'The Structure of Atonal Music'*, p. 127.

78. The close connection between 6–Z6 (0,1,2,5,6,7) and 6–Z38 (0,1,2,3,7,8) in the first segment reflects a unique state of affairs, since these set classes represent the only case of Z-paired hexachords being fashioned from combination interval cycles (all other Z-hexachords incorporated into Table 5 are lacking their Z-partners). Reference to Table 5 reveals that the cycle of semitones a tritone apart (interval-1 (5,6) (6,7)) and the cycle of fourths a tritone apart (interval-5 (1,6) (6,11)) both generate set classes 3–5 (0,1,6), 4–8 (0,1,5,6), 4–9 (0,1,6,7) and 5–7 (0,1,2,6,7) as contiguous events; only at cardinal 6 do they diverge, with set classes 6–7 (0,1,2,6,7,8) and 6–Z6 (0,1,2,5,6,7) in the former, and 6–7 (0,1,2,6,7,8) and 6–Z38 (0,1,2,3,7,8) in the latter. We could say that the instances of 6–Z6 (A♭, A, B♭, C♯, D, E♭) and 6–Z38 (A♭, A, C♯, D, E♭, E)

show a minimal perturbation from their notional progenitor 6–7 (G, A♭, A, C♯, D, E♭), requiring the transposition or displacement of just one element (G) by a minor third in each case. Interestingly, 6–Z6 and 6–Z38 are the only hexachords other than all-combinatorial ones to have a low occurrence of membership in Forte's system of genera, appearing in only two genera: this is clearly a consequence of their close •(R_2, R_P) relational similarity to 6–7, which also only occurs in the same two genera: see Forte, *The Structure of Atonal Music*, p. 185.

79. Cohn, 'Maximally Smooth Cycles', pp. 33–4.

80. Forte, 'Afterword', p. 242.

81. This seems to me to be easier than contracting six (genus) dimensions into three.

82. The M_5 transform allows all Z hexachords to be correlated transformationally, some interchangeably in groups of four and some in pairs, in alternating Z-paired and M_5-paired cyclic patterns:

 (1) 6–Z3 (0,1,2,3,5,6) Z-relates to 6–Z36 (0,1,2,3,4,7), which M-relates to 6–Z47 (0,1,2,4,7,9), which Z-relates to 6–Z25 (0,1,3,5,6,8), which M-relates back to 6–Z3 (alternating Chromatic and Diatonic set classes);
 (2) 6–Z4 (0,1,2,4,5,6) Z-relates to 6–Z37 (0,1,2,3,4,8), which M-relates to 6–Z48 (0,1,2,5,7,9), which Z-relates to 6–Z26 (0,1,3,5,7,8), which M-relates back to 6–Z4 (alternating Chromatic and Diatonic set classes);
 (3) 6–Z10 (0,1,3,4,5,7) Z-relates to 6–Z39 (0,2,3,4,5,8), which M-relates to 6–Z24 (0,1,3,4,6,8), which Z-relates to 6–Z46 (0,1,2,4,6,9), which M-relates back to 6–Z10 (alternating Chromatic and Diatonic set classes);
 (4) 6–Z13 (0,1,3,4,6,7) Z-relates to 6–Z42 (0,1,2,3,6,9), which M-relates to 6–Z29 (0,1,3,6,8,9), which Z-relates to 6–Z50 (0,1,4,6,7,9), which M-relates back to 6–Z13 (Octatonic set classes);
 (5) 6–Z23 (0,2,3,5,6,8) Z-relates to 6–Z45 (0,2,3,4,6,9), which M-relates to 6–Z45, which Z-relates to 6–Z23, which M-relates back to 6–Z23 (Octatonic set classes);
 (6) 6–Z28 (0,1,3,5,6,9) Z-relates to 6–Z49 (0,1,3,4,7,9), which M-relates to 6–Z49, which Z-relates to 6–Z28, which M-relates back to 6–Z28 (Octatonic set classes);
 (7) 6–Z19 (0,1,3,4,7,8) Z-relates and M-relates to 6–Z44 (0,1,2,5,6,9), which Z-relates and M-relates back to 6–Z19 (Hexatonic set classes);
 (8) 6–Z6 (0,1,2,5,6,7) Z-relates and M-relates to 6–Z38 (0,1,2,3,7,8), which Z-relates and M-relates back to 6–Z6 (Bichromatic set classes); and

(9) 6–Z11 (0,1,2,4,5,7) Z-relates and M-relates to 6–Z40 (0,1,2,3,5,8), which Z-relates and M-relates back to 6–Z11 (Chromatic and Diatonic set classes).

83. Transpositional cycles become altered when an additional operation such as T_nI is imposed on the M operation or on the individual set classes. We can see this occurring between instances of another Z/M-related pair, 4–Z15 (0,1,4,6) and 4–Z29 (0,1,3,7), towards the end of the fourth of Berg's *Four Songs* for voice and piano, Op. 2 ('Warm die Lüfte'), in bars 20–22. This coda progression can be portrayed as an augmented-ninth to dominant-thirteenth cycle, incorporating linear fourths in the bass and descending semitones in the upper three parts, and is based closely on a whole-tone progression which has already appeared (in the key of E♭ minor) at the start and close of the second Op. 2 song, and on chords used previously during bars 5–10 of the first song. The progression from the second song implies M_7,T_{11}/T_5 ic 1/ic 5-cycle transforms between whole-tone chords. That from the fourth song involves a series of T_nI, M_7, T_{11}/T_5 ic 1/ic 5-cycle transforms between 4–Z29s (0,1,3,7) and 4–Z15s (0,1,4,6), incorporating transpositions at T_{10} (T_{11} + T_{11} or T_5 + T_5) between instances of each set. The M_7,T_{11}/T_5 aspect of both sequences allows them to be viewed two ways, either as three parallel upper voices falling in semitones against a bass rising in fourths (as written by Berg), or as three parallel lower voices rising in fourths against a treble falling in semitones.

84. These ten correlations correspond to Gollin's ten non-replicating M cycles and Straus's ten i-chain pairs: see n. 46.

85. Additionally, in Table 8 the set classes labelled as subsets of the genus-defining set (in bold), as exclusive to the genus (*) or as gregarious (+) are similarly matched M-relationally within Hexatonic, Bichromatic, Whole-Tone and Octatonic lists, and are either opposed one-to-one or correlated in the Diatonic and Chromatic. Quinn has noted the M_5/M_7 correlations between chromatic and diatonic set classes in Hanson's projections, Eriksson's maxpoints and regions and Buchler's interval-saturated sets: see Quinn, 'General Equal-Tempered Harmony (Introduction and Part 1)', p. 138. The M relation also informs the symmetries in Eriksson's ex. 11 and Hoffman's figs 6 and 8, while the latter's figs 7a and 7c are M transforms of each other: see Eriksson, 'The IC Max Point Structure', p. 107; and Hoffman, 'On Pitch-Class Set Cartography', pp. 230, 232, 234 and 236.

86. Forte, 'Pitch-Class Set Genera', pp. 200, 204 and 218.

87. *Ibid.*, p. 215. His table 17 shows the distribution of four-, five- and six-element subsets of the diatonic collection, set class 7–35

(0,1,3,5,6,8,10), in relation to his genera, while his table 19 shows the chromatic collection, set class 7–1 (0,1,2,3,4,5,6), in the same way (pp. 212 and 216). Inevitably, both lists contain the same number of set classes, and each is a complete and exact match onto the other via the M_5 transform. Some matched set classes are shared between the same genera (i.e. genera 1, 2, 3 and 7), while others are paired across correlated but dichotomous chromatic-versus-diatonic genera (i.e. genus 5 versus genus 11, genus 6 versus genus 12 and genus 8 versus genus 10). In particular, his genera 6 ('semichroma') and 12 ('dia-tonal') are M_5-matched to each other. Forte might perhaps have named them relationally had he realised the correlation. Likewise, genus 8 might better have been called 'atonal-chroma' rather than simply 'atonal', thus corresponding to the 'atonal-tonal' of the associated, partially M_5-related genus 10 (Forte's genera 5 and 11 have already aptly been called 'chroma' and 'dia'). Indeed, Forte was close to realising these similarities when he produced identical difference quotients for his Supragenus II (chromatic)/Supragenus III (atonal) versus Supragenus III (atonal)/ Supragenus IV (diatonic), and also for his Supragenus I (atonal hybrid)/ Supragenus II (chromatic) versus Supragenus I (atonal hybrid)/ Supragenus IV (diatonic); see Forte, 'Pitch-Class Set Genera', p. 228, table 25. In his interpretation of these 'somewhat unexpected' difference quotient results, Forte makes the assumption that Supragenera II and IV (the chromatic and diatonic) both contribute (equally) to the structure of Supragenera I and III (the atonal hybrid and atonal): *ibid.*, p. 227. Although he notes the regularity in the counts of sets of the same cardinality between supragenera, he does not mention that the cycle of fourths/fifths M_5 transform might be implicated. He concludes this section of text by stating that '[f]urther study of the structure of the system of 12 pitch-class genera may reveal the significance of these evidently special circumstances': *ibid.*, pp. 227–9.

88. His chosen trichordal progenitors confirm this. Outside the spectrum are genera formed from the single 'atonal' trichords: 3–5 (0,1,6) and 3–8 (0,2,6) initiate genera 1 and 2, which have a set-class structure akin to my Bichromatic; 3–10 (0,3,6) initiates genus 3, which is akin to my Octatonic; and 3–12 (0,4,8) initiates genus 4, which relates to my Hexatonic. More significant, *within* the spectrum are genera formed from pairs of diatonic and chromatic trichords, which can be arranged in a symmetrical progression. Unsurprisingly, this progression transpires to be largely (pro)created by the chromatic-to-diatonic M_5-transform pairs, 3–1/3–9 (0,1,2)/(0,2,7), 3–2/3–7 (0,1,3)/(0,2,5) and 3–3/3–11 (0,1,4)/(0,3,7). Forte makes brief mention of this; *ibid.*, p. 267, n. 5.

89. According to Clampitt, the composer and theorist Anatol Vieru 'views the diatonic-chromatic duality as a fundamental aspect of the 12-pc universe';

see David Clampitt, 'Report: an International Symposium on Music and Mathematics (Bucharest, Romania)', *Music Theory Online*, 1/i (1995), para. 3. Quinn has debated whether the diatonic and chromatic genera are 'categorically opposite to one another' to the extent that they '*constrain* one another' but ultimately opts for a 'constrained independence' between them, which does not necessarily have to be musically opposed: see Quinn, 'General Equal-Tempered Harmony: Parts 2 and 3', pp. 53 and 55; emphasis in original. He nevertheless points out various intergeneric aspects of this symmetry: 'The degree to which a species [set class] ... exemplifies one genus [diatonic or chromatic] is the same as the degree to which its M_5-transform exemplifies the other genus [chromatic or diatonic]', and 'whatever an inclusional similarity relation says about species [set classes] *s* and *t* will be the same as what it says about M_5s and M_5t'; see Quinn, 'General Equal-Tempered Harmony (Introduction and Part 1)', pp. 138, 143 and 146. This feature is amply exemplified in the BIG chains of Tables A1–A6.

90. Quinn, 'General Equal-Tempered Harmony (Introduction and Part 1)', p. 146; emphasis in original.

91. Quinn has pointed out that the all-combinatorial set class 6–8 (0,2,3,4,5,7) has characteristics of both the diatonic and chromatic; Eriksson has included this set class in his diatonic-with-chromatic region 7, along with the diatonic and chromatic set classes 6–9 (0,1,2,3,5,7), 6–Z11 (0,1,2,4,5,7), 6–Z40 (0,1,2,3,5,8), 5–11 (0,2,3,4,7), 4–11 (0,1,3,5) and 4–10 (0,2,3,5). See Quinn, 'General Equal-Tempered Harmony: Parts 2 and 3', p. 55; and Eriksson, 'The IC Max Point Structure', pp. 102–3, ex. 7. These seven set classes are distributed into both genera in my Table 8.

92. This overarching diatonic-to-chromatic spectrum contrasts neatly with Forte's diatonic-to-exotic spectrum.

93. Samplaski has only incorporated set classes of cardinals 3, 4 and 5 in these histograms; Samplaski, 'Mapping the Geometries', paras 36 and 43, and figs 7 and 11. His fig. 7, labelled 'ANGLE', represents an interval-class saturation arrangement derived from Isaacson, 'Similarity of Interval-Class Content', while his fig. 11, labelled 'RECREL', embodies a subset-embedding array drawn from Castrén, 'RECREL: a Similarity Measure'. Both are computationally induced.

94. 'All of the set-classes in the [central near-zero] clump have equal ic1 and ic5 content; the first few set-classes with significantly non-zero coordinates [to either side of the central clump] have ic1 content one more than ic5 content or vice versa; and the set-classes at the extremes have either zero ic5 or ic1 content while the value of the opposite is at least two ... .

This dimension thus provides a graphic ... example of ... what the terms "diatonic" and "chromatic" really signify'. Samplaski, 'Mapping the Geometries', para. 39.

95. It is worth noting that the characteristic set classes for some of my genera are also clustered to some extent in Samplaski's scalings, with the Bichromatic set classes somewhat closer to the chromatic nexus and the Hexatonic ones somewhat closer to the diatonic.

96. His fig. 9, however, is based on ANGLE coordinates for a whole-tone–versus–anti-whole-tone dimension, although the resultant spread runs from M-invariant whole-tone set classes at one extreme to octatonic and/or bichromatic M invariants at the other, with the neutral 4–Z29 at its centre; his fig. 8 is based on ANGLE coordinates for set class 3–5 (0,1,6), i.e. bichromatic, versus interval class 3, i.e. nominally octatonic, but more appositely anti-bichromatic, which displays a spread useful for the Bichromatic genus, again with set class 4–Z29 at its centre.

97. I have decided at this point to drop the low ic 6 attribute as unnecessary to the process, since it is typical of both Diatonic and Chromatic genera.

98. In the case of Octatonic scalings, where a different number of interval-class ratings is being compared, cardinalities below 6 have needed to be promoted in the rankings, pentachords by one, tetrachords by two, etc., in order to create a proper spread across the cardinalities.

99. Specifically, with the Diatonic and Chromatic spectrum, since the difference between ics 1 and 5, as shown by the number of pluses for all set classes of cardinals 2–6 in Table 9, ranges from four to minus four, it would be convenient to say that there are nine distinct levels that are reduced to seven by conflating the top two and bottom two levels. The most typical set classes could then be labelled '1' (i.e. those with a positive 1/5 difference scoring of 4 or 3), the next level could be labelled '2' for a difference scoring of 2, and so on down to '7' for a negative scoring of 4 or 3 (i.e. the least typical set classes). It is worth reiterating here that the chromatic range is different in kind to Quinn's chromatic array since it does not follow a maximally chromatic to maximally even dimension; see Quinn, 'General Equal-Tempered Harmony (Introduction and Part 1)', p. 47, ex. 43. Further, the uppermost five of the Hexatonic's ten affinity levels have been reduced to two and level six has been omitted in order to present the neutral set classes centrally as level '4'; the Bichromatic's fifteen levels have been grouped in twos or threes; the Whole-Tone's ten levels have likewise been reduced to seven by conflating the uppermost five to two; and finally, the Octatonic's fourteen levels have been reduced to seven by conjoining the top twelve levels, grouping them in threes or twos.

100. If we look at Samplaski's seven other histograms, which show a variety of coordinated parameters measuring similarity/dissimilarity relations other

than the diatonic-versus-chromatic discussed earlier, we find that all 24 M-related set classes of cardinals 3–5 once again pervade the structures, although now forming *adjacent* pairs within the distributions (the few exceptions being almost adjacently paired). This remarkable phenomenon presumably occurs because, whenever oppositions other than diatonic-versus-chromatic are under consideration, the M-related sets inevitably demand to sit snugly together as a pair through the twinning of their ic 2, 3, 4 and 6 content. This situation seems to correlate with the consistent placement of these pairs together as symmetrical oppositions in the Diatonic and Chromatic genera but as related pairs in the other four genera, as can be seen in Table 8.

101. Unlike his chromatic range (see n. 99), Quinn's (incomplete) hexatonic and whole-tone ranges *do* concur with those in Table 10; see Quinn, 'General Equal-Tempered Harmony (Introduction and Part 1)', p. 48, ex. 44.

102. Some examples are 6–Z12/6–Z41 [332232], 6–Z17/6–Z43 [322332], 6–Z11/6–Z40 [333231], 6–8 [343230], 5–11 [222220], 5–Z12/5–Z36 [222121], 5–15 [220222], 5–Z18/5–Z38 [212221], 5–19 [212122], 5–28 [122212], 4–5 [210111], 4–16 [110121], 4–11 [121110], 4–13 [112011] and 4–Z15/4–Z29 [111111].

103. Tenkanen, 'A Linear Algebraic Approach', p. 527.

104. These three areas are, firstly, a block at one extreme containing his region 6 (high ic 6 content, corresponding to my Bichromatic); secondly, a block at the other extreme containing his regions 3 (high ic 3 and 6 content, corresponding my Octatonic) and 2 (high ic 2, 4 and 6 content, corresponding to my Whole-Tone); and thirdly, a centrally situated block containing his regions 1 (high ic 1 content, corresponding to my Chromatic), 5 (high ic 5 content, corresponding to my Diatonic), 4 (high ic 4 content, corresponding to my Hexatonic) and 7 (high ic 2 content, an M_5 self-mapping overlap between his regions 1 and 5); see Eriksson, 'The IC Max Point Structure', p. 105, ex. 9.

105. This merges a chromatic/diatonic area (the M_5-paired pentachords 5–1/5–35 (0,1,2,3,4)/(0,2,4,7,9), 5–2/5–23 (0,1,2,3,5)/(0,2,3,5,7) and 5–3/5–27 (0,1,2,4,5)/(0,1,3,5,8)) with a whole-tone area (the singleton set class 5–33 (0,2,4,6,8)) via a chromatic/diatonic/whole-tone mix (the M_5-paired 5–9/5–24 (0,1,2,4,6)/(0,1,3,5,7)); see Quinn, 'General Equal-Tempered Harmony (Introduction and Part 1)', p. 147, ex. 11. These set classes also conform appropriately to my genera, with 5–9/5–24 in all three.

106. This comprises set classes 4–1 (0,1,2,3), 4–2 (0,1,2,4), 4–11 (0,1,3,5), 5–24 (0,1,3,5,7), 5–23 (0,2,3,5,7), 7–27 (0,1,2,4,5,7,9) and 7–35 (0,1,3,5,6,8,10); see Quinn, 'General Equal-Tempered Harmony: Parts 2

and 3', p. 50, ex. 44. This graph has perpendicular axes representing the 'non-chromatic to most-chromatic' and the 'non-diatonic to most-diatonic' aspects of the transition and a central M-invariant diagonal axis representing a 'non-chromatic-or-diatonic to most-chromatic-*and*-diatonic' aspect. Again, the central set classes in the sequence, 4–11 and 5–24, are both diatonic and chromatic.

107. This progression runs from 6–20 (0,1,4,5,8,9), which has six instances of ic 4 within its ic vector, via 6–Z19 (0,1,3,4,7,8), 6–Z44 (0,1,2,5,6,9), 6–14 (0,1,3,4,5,8), 6–15 (0,1,2,4,5,8), 6–31 (0,1,3,5,8,9), 6–16 (0,1,4, 5,6,8), 6–21 (0,2,3,4,6,8), 6–34 (0,1,3,5,7,9) and 6–22 (0,1,2,4,6,8), all with four instances of ic 4, and terminating at 6–35 (0,2,4,6,8,10), which again has six instances of ic 4; see Quinn, 'General Equal-Tempered Harmony: Parts 2 and 3', p. 54, ex. 48. Note that the hexachords in this third model replicate those presented in my Hexatonic and Whole-Tone families, Table 6.

108. Callender, Quinn and Tymoczko, 'Generalized Voice-Leading Spaces', fig. S10 and fig. S5, A–H; and Straus, 'Voice Leading in Set-Class Space'.

109. Richard Cohn, 'A Tetrahedral Graph of Tetrachordal Voice-Leading Space', *Music Theory Online*, 9/iv (2003), fig. 10; and Callender, Quinn and Tymoczko, 'Generalized Voice-Leading Spaces', fig. S5, L–M.

110. Atte Tenkanen, 'Measuring Tonal Articulations in Compositions', paper given at the MaMuX Computational Analysis Special Session, at IRCAM, Paris, April 2008, p. 7, http://recherche.ircam.fr/equipes/repmus/mamux/Tenkanen.pdf.

111. Cohn, 'A Tetrahedral Graph', p. 9.

112. Quinn, 'General Equal-Tempered Harmony (Introduction and Part 1)', p. 146.

113. *Ibid.*, p. 146–7; emphasis in original. In fact, all M-invariant set classes are captured by my four non-Diatonic/Chromatic genera.

114. Cohn, 'Maximally Smooth Cycles', pp. 31 and 34.

115. In other words, all six genus prototypes would be situated 'at the "edges" of the distribution'; see Quinn, 'General Equal-Tempered Harmony (Introduction and Part 1)', p. 135.

116. An octahedron or square dipyramid could be used as an alternative to the sphere. It is worth noting here that the smaller (TINV) progenitors of the four 'equatorial' genera, 3–12 (0,4,8), 4–9 (0,1,6,7), 4–25 (0,2,6,8) and 4–28 (0,3,6,9), have no place in the Diatonic and Chromatic genera and therefore contribute to the definition of an essentially different, M-invariant, axis of generation.

117. This arrangement is similar to Eriksson's regional layout, discussed earlier, in that a bichromatic/hexatonic axis embraces a diatonic/chromatic one; see Eriksson, 'The IC Max Point Structure', p. 105, ex. 9. Quinn has a similar axial arrangement of diatonic/octatonic/chromatic against bichromatic/octatonic/hexatonic, with whole-tone as an isolate; see Quinn, 'Listening to Similarity Relations', p. 130, ex. 8.

118. Since the Whole-Tone region appears to be somewhat separate from the Hexatonic, Bichromatic and Octatonic, an alternative three-dimensional shape would have the Whole-Tone somewhat offset from, or even opposed to, the other three equatorial nodes. It could also be argued that the Bichromatic, Diatonic and Chromatic nodes should be a little more closely aligned, since these are the only three to be properly related to the full twelve-note aggregate (see the generational process of each in Table 5). If set class 6–27 (0,1,3,4,6,9) rather than set class 6–30 (0,1,3,6,7,9) had been chosen as the Octatonic progenitor, then this region would have leaned less towards the Bichromatic and slightly more towards the other three.

119. The choice of just three axes, rather than one per genus, is a practical one, although not without its own inherent problems, based on the desire to create a visually comprehensible representation in just three dimensions. For practical reasons, the diagram has had to be drawn in distorted form with the Hexatonic node apparently somewhat closer to the Whole-Tone than to the Octatonic and somewhat closer to the Diatonic than to the Chromatic, and the Bichromatic node apparently somewhat closer to the Octatonic than to the Whole-Tone and somewhat closer to the Chromatic than to the Diatonic. I am visually assuming the Hexatonic to be the closer node and the Bichromatic the farther away, although of course the viewer can equally imagine the whole diagram the other way round.

120. These are Diatonic to Hexatonic, Hexatonic to Chromatic, Chromatic to Bichromatic, Bichromatic to Diatonic, Diatonic to Whole-Tone, Whole-Tone to Chromatic, Chromatic to Octatonic, Octatonic to Diatonic, Whole-Tone to Hexatonic, Hexatonic to Octatonic, Octatonic to Bichromatic and Bichromatic to Whole-Tone.

121. The Diatonic/Chromatic, Whole-Tone/Octatonic and Hexatonic/Bichromatic coordinates occupy the same point, in the centre of the globe; the last is not shown in Fig. 6, as it happens to represent an empty two-genus node.

122. The only nodal points missing on the horizontal planes shown in Figs 7a, b and c are the Diatonic and Chromatic S/N extremes.

123. Quinn, 'General Equal-Tempered Harmony: Parts 2 and 3', pp. 30 and 50.

124. *Ibid.*, p. 54, ex. 48.

125. It needs to be understood, however, that the two remaining two-genus interfaces at this central point bypass each other without having the apparent closeness suggested by the diagram, and that the six-genus interface (set class 2–4) necessarily has to placed adjacent to them as well.

126. This total of 6 plus 57 would be a round 64 if we were to include the zero-rated category outside the global set-class space.

127. Quinn, 'General Equal-Tempered Harmony (Introduction and Part 1)', p. 135.

128. By implication, their complements would occupy these same positions, but they have been omitted for reasons of space and clarity.

129. Quinn, 'General Equal-Tempered Harmony: Parts 2 and 3', p. 30.

130. This same arrangement applies within the central near-zero 'clump' in Samplaski's figs 7 and 11.

131. Quinn's three two-dimensional axes (nn. 105, 106 and 107) can clearly be seen projected within Fig. 8. Firstly, his triangular diatonic/chromatic/whole-tone group, set classes 5–1/5–35 (0,1,2,3,4)/(0,2,4,7,9), 5–2/5–23 (0,1,2,3,5)/(0,2,3,5,7), 5–3/5–27 (0,1,2,4,5)/(0,1,3,5,8) and 5–33 (0,2,4,6,8) is duly projected as an arc on the left-hand (Whole-Tone) side of centre. Secondly, his chromatic-to-diatonic Cartesian plane sequence, set classes 4–1 (0,1,2,3), 4–2 (0,1,2,4), 4–11 (0,1,3,5), 5–24 (0,1,3,5,7), 5–23 (0,2,3,5,7), 7–27 (0,1,2,4,5,7,9) and 7–35 (0,1,3,5,6,8,10), follows as a north-to-south continuum, deviating somewhat towards the Whole-Tone during its course. Thirdly, his hexatonic-to-whole-tone progression of hexachords, set classes 6–20 (0,1,4,5,8,9), 6–Z19 (0,1,3,4,7,8), 6–Z44 (0,1,2,5,6,9), 6–14 (0,1,3,4,5,8), 6–15 (0,1,2,4,5,8), 6–31 (0,1,3,5,8,9), 6–16 (0,1,4,5,6,8), 6–21 (0,2,3,4,6,8), 6–34 (0,1,3,5,7,9), 6–22 (0,1,2,4,6,8) and 6–35 (0,2,4,6,8,10), appears as a diverging and somewhat roundabout trip 'eastwards' from the Hexatonic node to the Whole-Tone node.

132. Tenkanen, 'A Linear Algebraic Approach', pp. 525–7; Samplaski, 'Mapping the Geometries', paras 66–7; and Forte, 'Pitch-Class Set Genera', p. 196.

133. Tenkanen, 'A Linear Algebraic Approach', p. 525; and Forte, 'Pitch-Class Set Genera', p. 192. Quinn keeps the whole-tone category unconnected to the other five in his illustration of tetrachord set classes under ASIM; see Quinn, 'Listening to Similarity Relations', p. 128, ex. 7, and p. 130, ex. 8.

134. Forte, 'Pitch-Class Set Genera', p. 196, states that it is 'exceedingly difficult to find any musical instantiation whatsoever' for such sets.

135. The 'no bias' collection does, however, include some sets with an equally weighted membership of both the Diatonic and the Chromatic genera.

Appendix: BIG Chains

Table A1 Hexatonic BIG chains

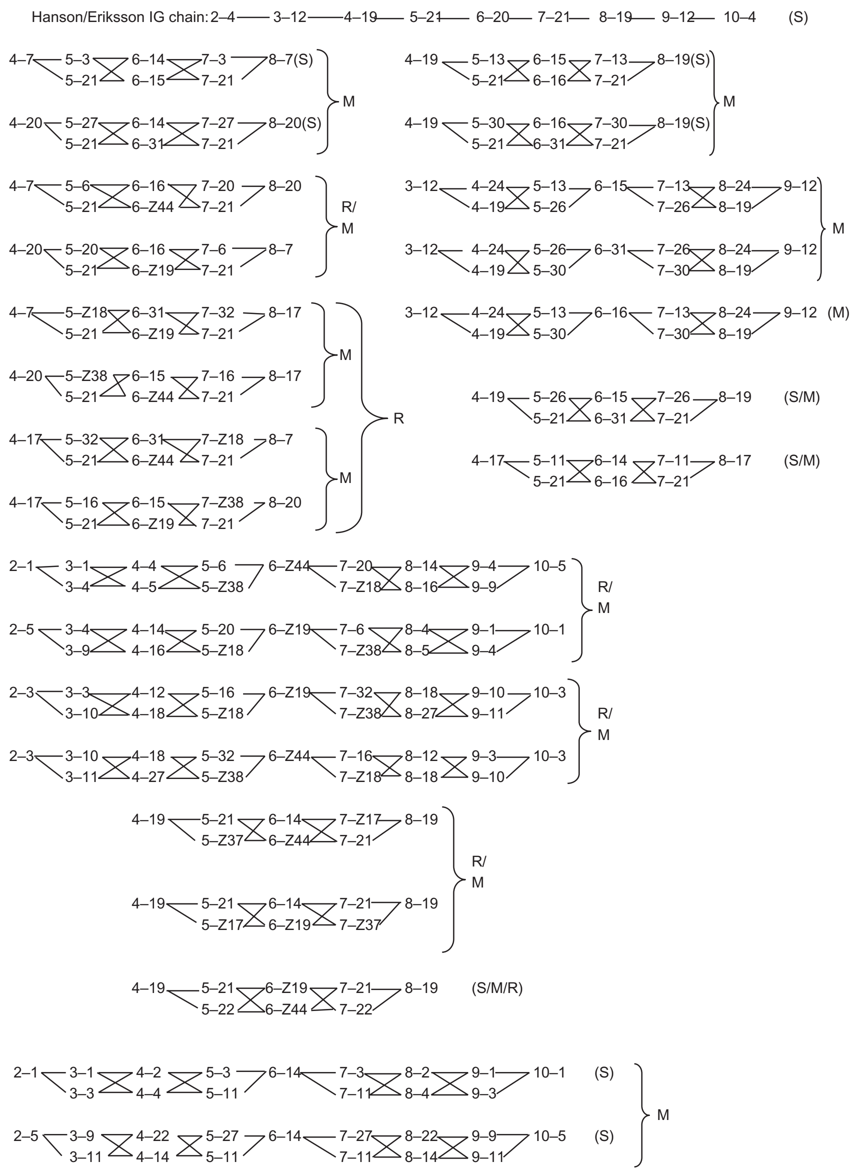


Table A2 Bichromatic BIG chains

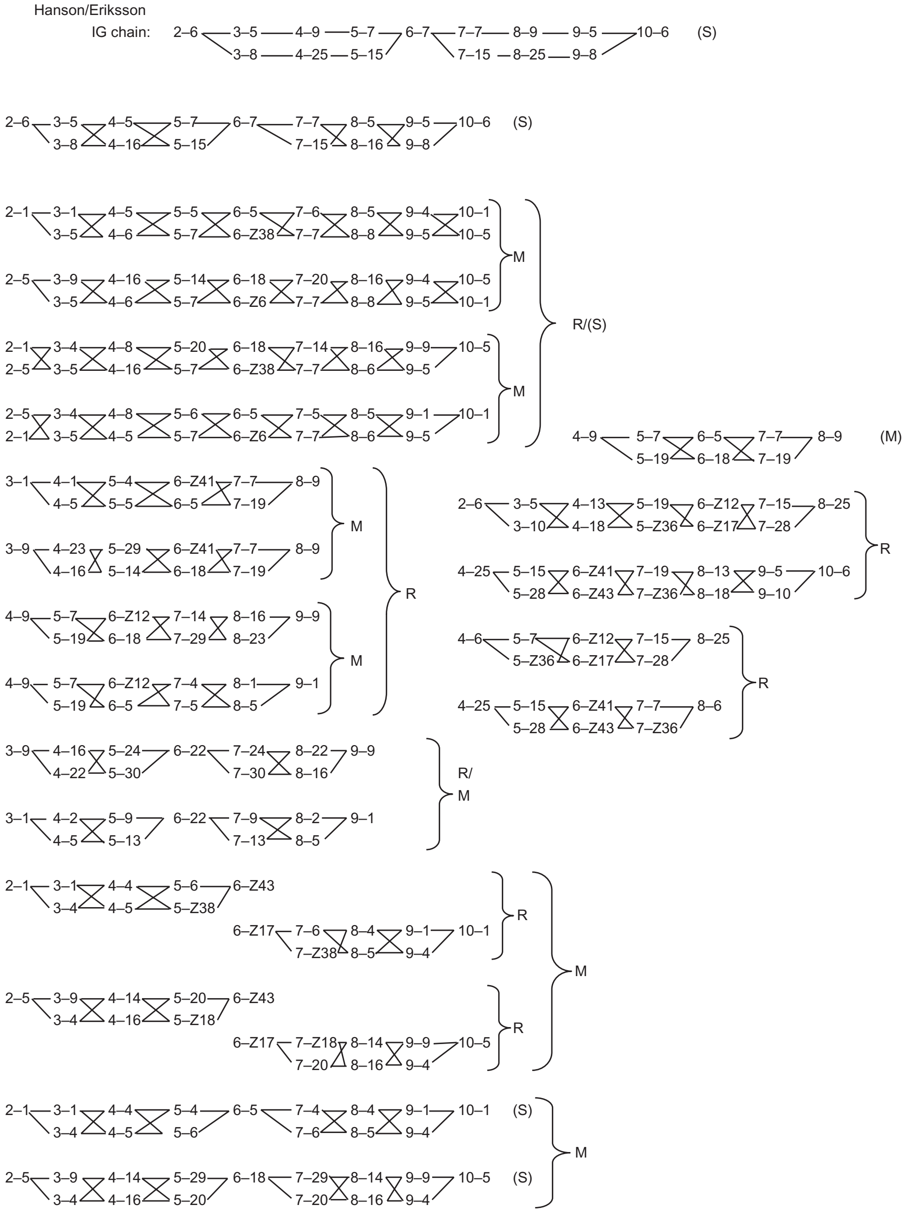


Table A3 Octatonic BIG chains

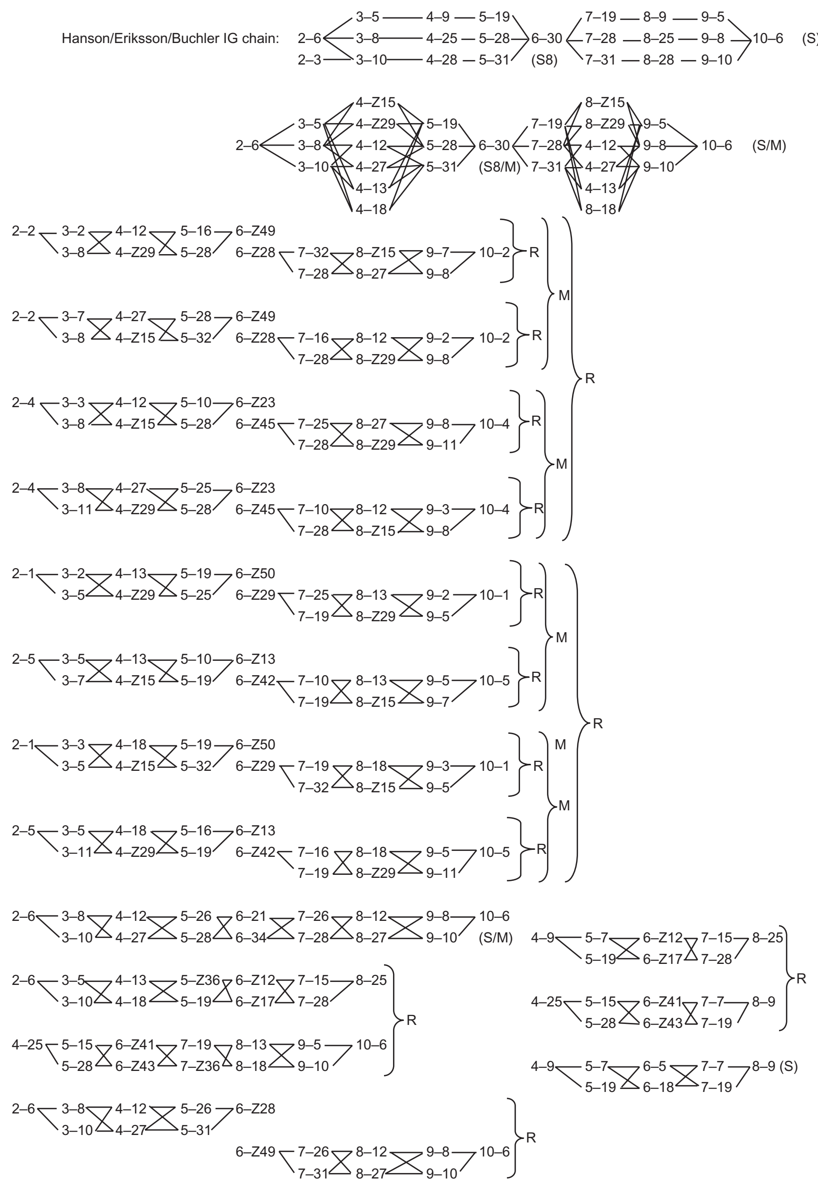


Table A3 Continued

Table A4 Whole-Tone BIG chains

Hanson/Eriksson/Buchler IG chain:

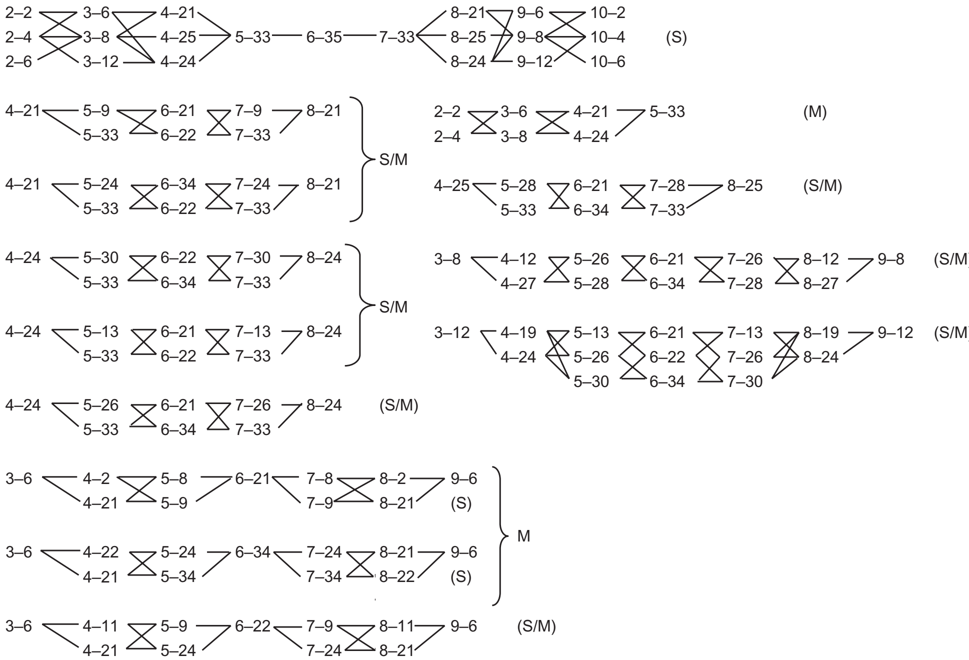


Table A5 Diatonic BIG chains

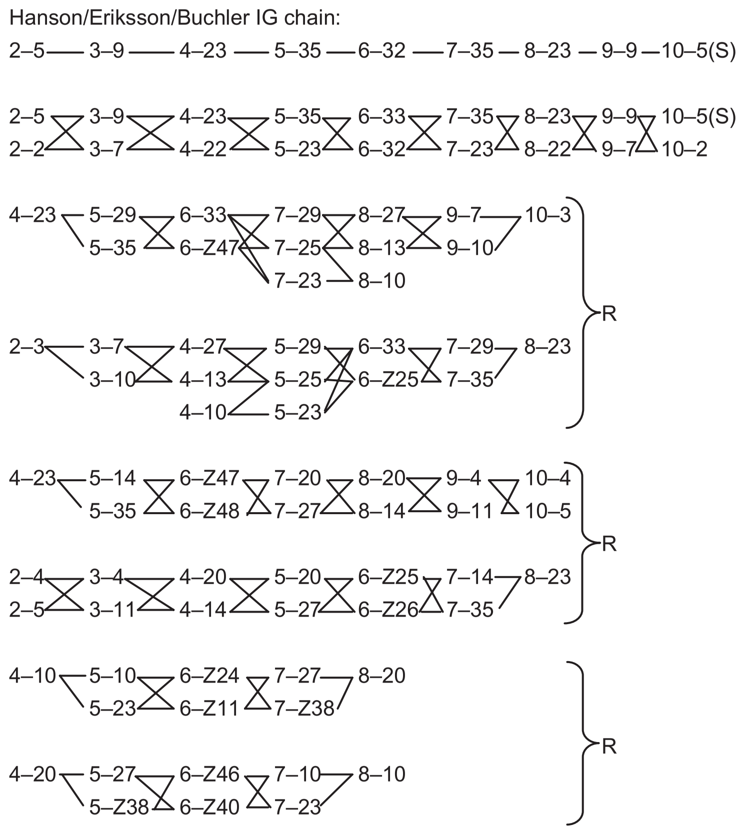


Table A6 Chromatic BIG chains

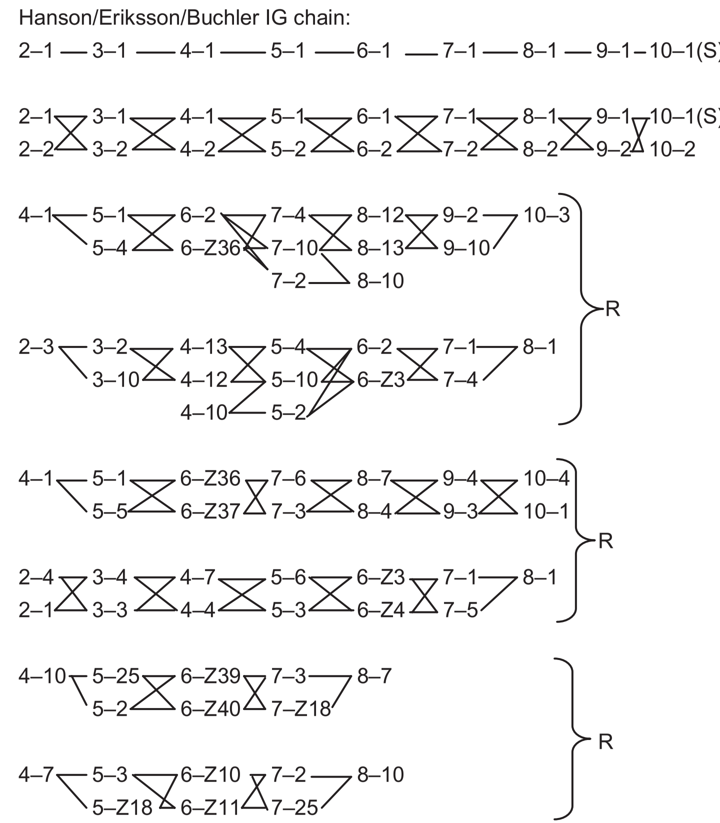
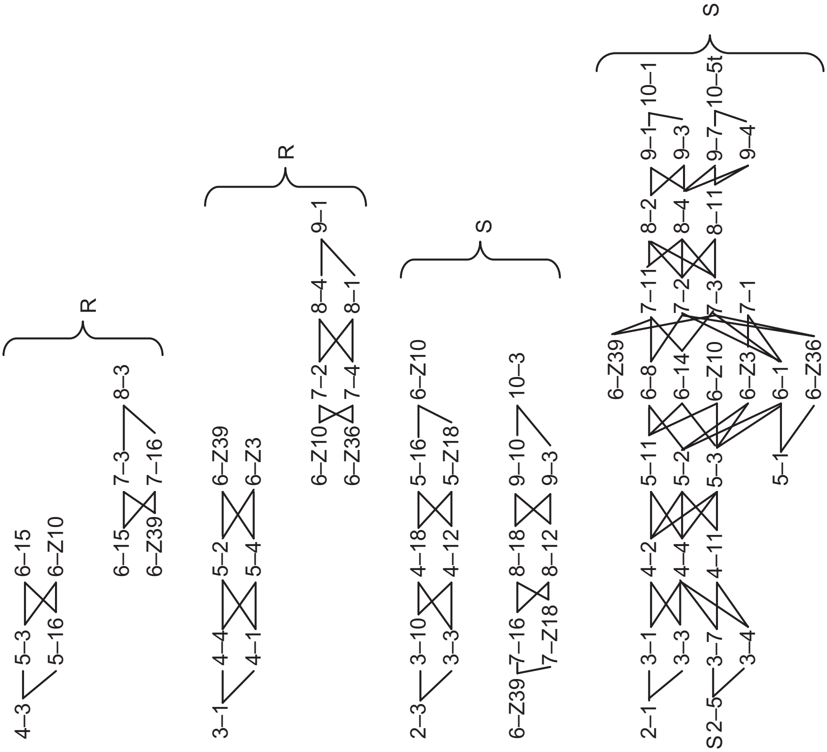


Table A6 Continued



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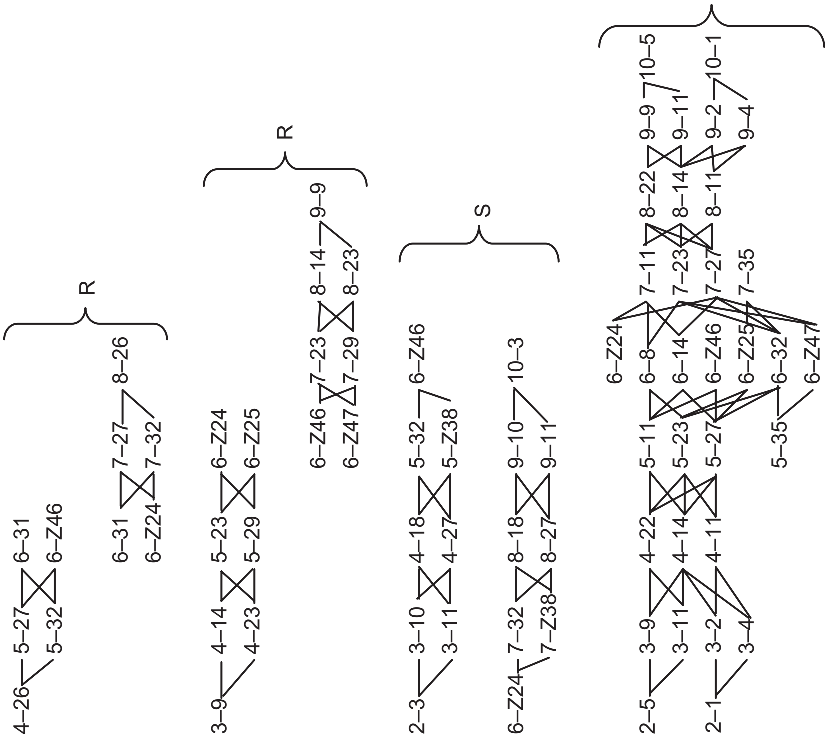

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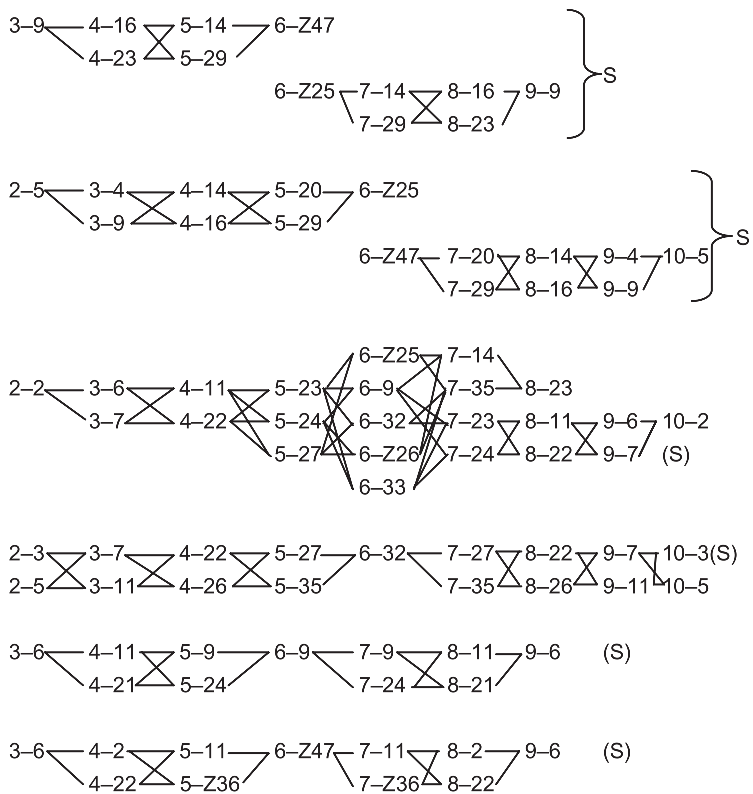
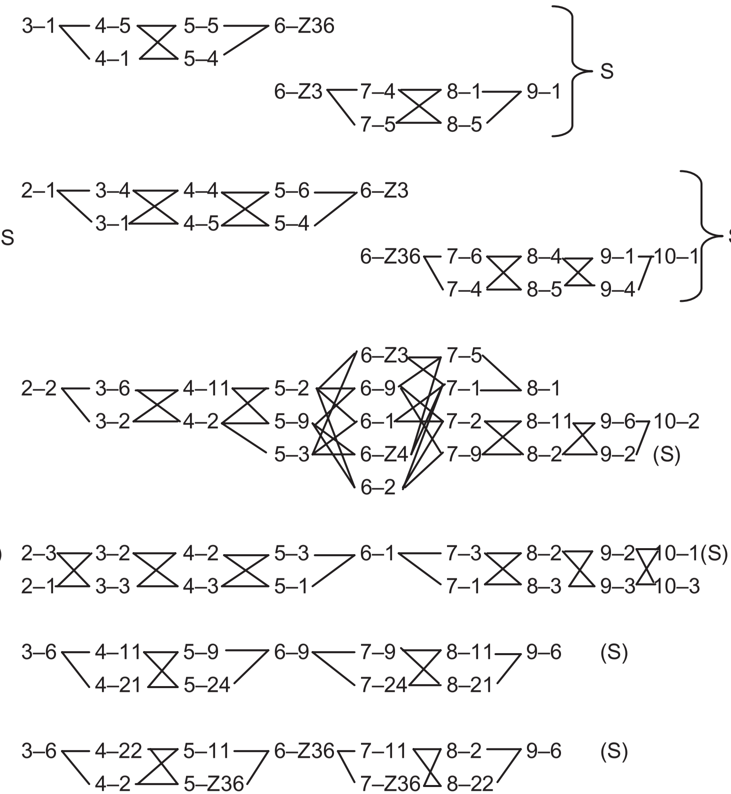


Table A6 Continued



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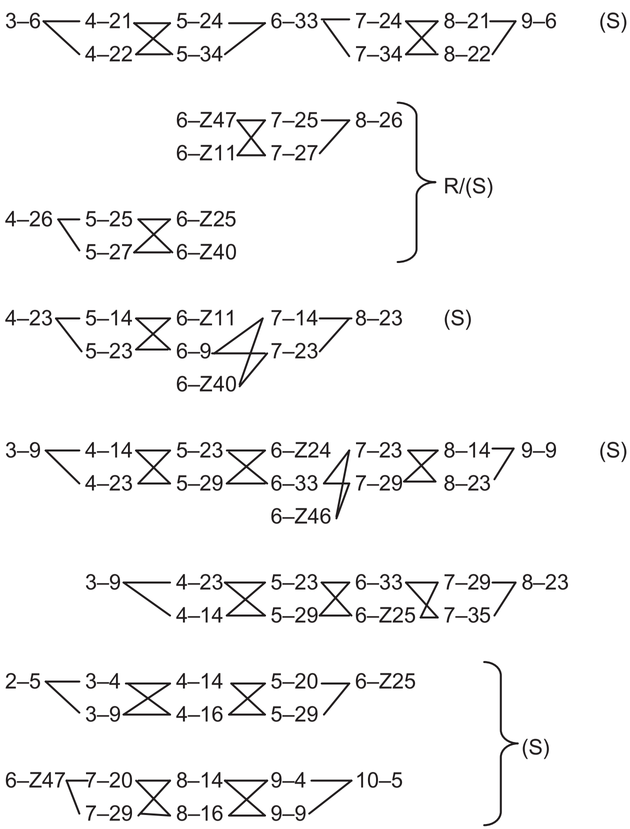
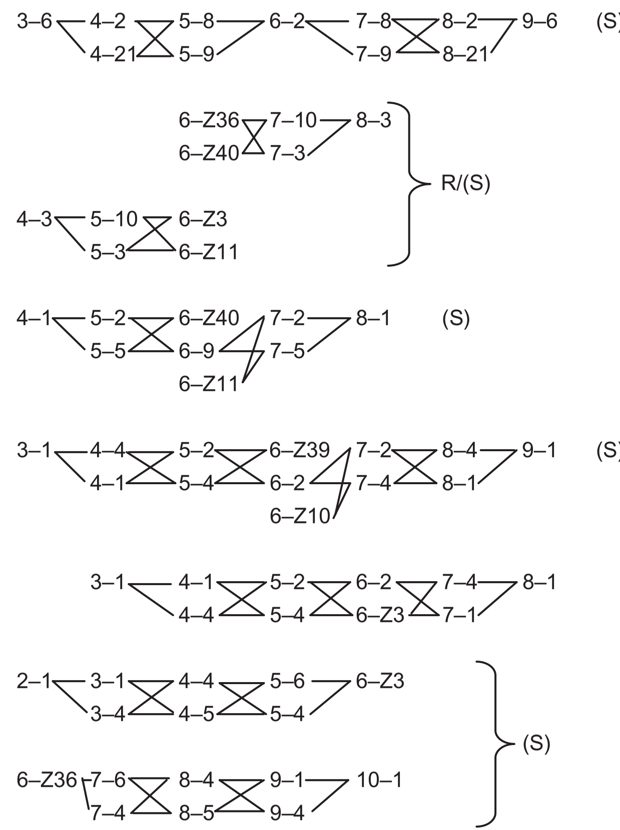


Table A6 Continued



NOTE ON CONTRIBUTOR

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ABSTRACT

The notion that pitch-class sets might occupy some form of multi-dimensional space has engaged a number of theorists over the years. The 'space odyssey' in this article delves into several new issues, such as a methodology for the creation of a comprehensive array of bonded inclusional growth chains, the formation of a diatonic-to-chromatic, M_5-induced spectrum of symmetrically arranged set classes and a simple method for establishing levels of affinity between set classes. These investigations lead to the advancement of a sixfold system of genera and to hierarchical distributions of all set classes, furnishing a distinct shape or profile to the whole set-class universe and to each of the genera. A productive way of producing a three-dimensional model incorporating all set classes presents itself if four foci, the 'bichromatic' 6–7 (0,1,2,6,7,8), hexatonic 6–20 (0,1,4,5,8,9), octatonic 6–30 (0,1,3,6,7,9) and whole-tone 6–35 (0,2,4,6,8,10), are granted their own spatial 'homes' at distant points on an M-invariant axis placed perpendicular to a diatonic 6–32 (0,2,4,5,7,9) to chromatic 6–1 (0,1,2,3,4,5) axis. A transparent globe would seem to be the most suitable vehicle for the representation of this arrangement, where the chromatic and diatonic foci take the 'north' and 'south' poles and the other four take their positions around the 'equator'.

CORRECTION STATEMENT

Corrections added on 11 February 2013, after first online publication on 7th January 2013: restoration of borders in Table 6; restoration of horizontal lines in Table 9; restoration of vertical lines in Table 10; removal of volume and issue numbers.